Classical gauge instantons from open strings

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Introduction

- Extend methods of field theory to strings
- For $\alpha' \to 0$ string theory gives gauge interactions + gravity
- Perturbative sector of field theories

$$\mathcal{A}_N = \int_\Sigma \langle V_{\phi_1} \cdots V_{\phi_N} \rangle_\Sigma$$

- Focus on the simplest geometry the sphere (closed strings) and the disk (open strings)
- Distinguish in the vertex $V_\phi = \phi\, \mathcal{V}_\phi$, the polarization ϕ from the operator part $\mathcal{V}_\phi$
- Then $\langle\, \mathcal{V}_{\phi\,\text{closed}}\, \rangle_{\text{sphere}} = \langle\, \mathcal{V}_{\phi\,\text{open}}\, \rangle_{\text{disk}} = 0$: no tadpoles!
- What about non perturbative backgrounds?

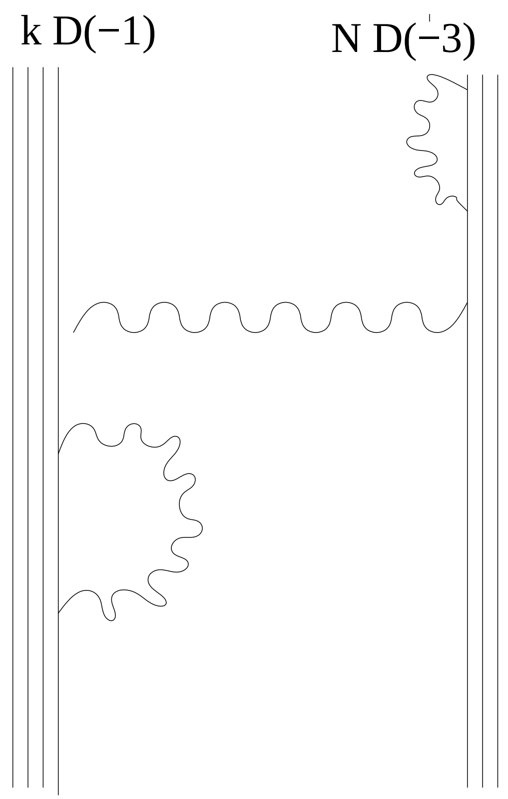
- In the presence of D-branes the simplest topology is the disk with (p+1) longitudinal and (9-p) transverse boundary conditions

- In this case $\langle\, \mathcal{V}_{\phi\,\mathrm{closed}}\,\rangle_{\mathrm{disk}_p} \neq 0$ or $\langle \phi\,\mathrm{closed} | \mathrm{D}p \rangle \neq 0$

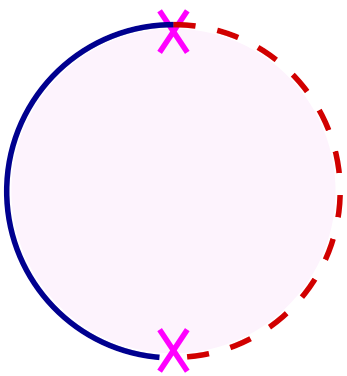
- $|\mathrm{D}p\rangle$ is the boundary state a classical source for the fields of the closed string spectrum

- For example $\langle\, \mathcal{V}_{h_{\mu\nu}}\,\rangle_{\mathrm{disk}_p} = \langle h_{\mu\nu} | \mathrm{D}p \rangle$ gives the metric of the Dp-brane in the large distance approx

- The system D(-1)/D3 gives the ADHM construction



- Different b.c. for the open string in the D(-1)/D3 imply the existence of “mixed” disks



- For mixed disks $\langle \mathcal{V}_{\phi_{\rm open}} \rangle_{\rm mixed\ disk} \neq 0$ and i will show that $\langle \mathcal{V}_{A_\mu} \rangle_{\rm mixed\ disk} \neq 0$

The D(-1)/D3 system

- In the D(-1)/D3 system the string coordinates $X^M(\tau,\sigma)$ and $\psi^M(\tau,\sigma)$ ($M=1,\ldots,10$)obey different b.c.
- The D(-1) has Dirichlet b.c. in all directions
- The D3 has Neumann for the longitudinal X^μ and ψ^μ ($\mu=1,2,3,4$) and Dirichlet for the transverse X^a and ψ^a ($a=5,\ldots,10$)
- The spin field $S^{\dot{\mathcal{A}}}$ is a Weyl spinor of $\mathrm{SO}(10)$. On the D(-1) boundary $S^{\dot{\mathcal{A}}}(z)=\widetilde{S}^{\dot{\mathcal{A}}}(\bar{z})\Big|_{z=\bar{z}}$ while on the D3 boundary $S^{\dot{\mathcal{A}}}(z)=\epsilon'\,(\Gamma^{0123}\widetilde{S})^{\dot{\mathcal{A}}}(\bar{z})\Big|_{z=\bar{z}}$

- The presence of the D3 breaks $\mathrm{SO}(10) \to \mathrm{SO}(4) \times \mathrm{SO}(6)$
- The D(-1) b.c. induce $S_\alpha(z)\, S_A(z) = \widetilde{S}_\alpha(\bar{z})\, \widetilde{S}_A(\bar{z})\Big|_{z=\bar{z}} \quad , \quad S^{\dot{\alpha}}(z)\, S^A(z) = \widetilde{S}^{\dot{\alpha}}(\bar{z})\, \widetilde{S}^A(\bar{z})\Big|_{z=\bar{z}}$
- The D3 b.c. induce $S_\alpha(z)\, S_A(z) = \varepsilon'\ \widetilde{S}_\alpha(\bar{z})\, \widetilde{S}_A(\bar{z})\Big|_{z=\bar{z}} \quad , \quad S^{\dot{\alpha}}(z)\, S^A(z) = -\varepsilon'\ \widetilde{S}^{\dot{\alpha}}(\bar{z})\, \widetilde{S}^A(\bar{z})\Big|_{z=\bar{z}}$

Broken and unbroken SUSY

- In the D(-1)/D3 system one can define bulk supercharges $Q^{\dot{\mathcal{A}}} = \frac{1}{2\pi\mathrm{i}}\int dz\; j^{\dot{\mathcal{A}}}(z)$ and $\widetilde{Q}^{\dot{\mathcal{A}}} = \frac{1}{2\pi\mathrm{i}}\int d\bar{z}\; \widetilde{j}^{\dot{\mathcal{A}}}(\bar{z})$
- $j^{\dot{\mathcal{A}}}$ ($\widetilde{j}^{\dot{\mathcal{A}}}$) is the left (right) SUSY current. In the $(-1/2)$ picture $j^{\dot{\mathcal{A}}}(z) = S^{\dot{\mathcal{A}}}(z)\,\mathrm{e}^{-\frac{1}{2}\phi(z)}$
- The b.c. $j(z) = \widetilde{j}(\bar{z})\Big|_{\bar{z}=z}$ conserves the charge $q = Q - \widetilde{Q}$ and breaks $q' = Q + \widetilde{Q}$

Summ ary

Charge	$D3$	$D(-1)$	Param
$Q^{\dot{\alpha}A} - \widetilde{Q}^{\dot{\alpha}A}$	u	u	$\bar{\xi}_{\dot{\alpha}A}$
$Q^{\dot{\alpha}A} + \widetilde{Q}^{\dot{\alpha}A}$	b	b	$\rho_{\dot{\alpha}A}$
$Q_{\alpha A} - \widetilde{Q}_{\alpha A}$	b	u	$\xi^{\alpha A}$
$Q_{\alpha A} + \widetilde{Q}_{\alpha A}$	u	b	$\eta^{\alpha A}$

Massless spectrum

- In the D(-1)/D3 system there are different open strings

i) (-1)/(-1)

ii) 3/3

iii) (-1)/3

iv) 3/(-1)

- In the 3/3 string

NS the gauge vector A^μ and six scalars φ^a which can propagate in the four longitudinal directions of the D3 brane

R two gauginos, $\Lambda^{\alpha A}$ and $\bar{\Lambda}_{\dot{\alpha} A}$

- The vertex operators are

$$V_A^{(-1)}(z) = A^\mu(p)\, \mathcal{V}_{A^\mu}^{(-1)}(z;p)$$
$$V_\varphi^{(-1)}(z) = \varphi^a(p)\, \mathcal{V}_{\varphi^a}^{(-1)}(z;p)$$
$$\mathcal{V}_{A^\mu}^{(-1)}(z;p) \;=\; \frac{1}{\sqrt{2}}\,\psi_\mu(z)\,\mathrm{e}^{-\phi(z)}\,\mathrm{e}^{\mathrm{i}p_\nu X^\nu(z)}$$
$$\mathcal{V}_{\varphi^a}^{(-1)}(z;p) \;=\; \frac{1}{\sqrt{2}}\,\psi_a(z)\,\mathrm{e}^{-\phi(z)}\,\mathrm{e}^{\mathrm{i}p_\nu X^\nu(z)}$$

- These fields form an $N=4$ vector multiplet

$$\delta A^{\mu} = \mathrm{i}\,\bar{\xi}_{\dot{\alpha}A}\,(\bar{\sigma}^{\mu})^{\dot{\alpha}\beta}\,\Lambda_{\beta}{}^{A} + \mathrm{i}\,\eta^{\alpha A}\,(\sigma^{\mu})_{\alpha\dot{\beta}}\,\bar{\Lambda}^{\dot{\beta}}{}_{A}$$

$$\delta \Lambda^{\alpha A} = \frac{\mathrm{i}}{2}\,\eta^{\beta A}\,(\sigma^{\mu\nu})_{\beta}{}^{\alpha}\,F_{\mu\nu} + \mathrm{i}\,\bar{\xi}_{\dot{\beta}B}\,(\bar{\sigma}^{\mu})^{\dot{\beta}\alpha}\,(\Sigma^{a})^{BA}\,\partial_{\mu}\varphi_{a}$$

$$\delta \bar{\Lambda}_{\dot{\alpha}A} = \frac{\mathrm{i}}{2}\,\bar{\xi}_{\dot{\beta}A}\,(\bar{\sigma}^{\mu\nu})^{\dot{\beta}}{}_{\dot{\alpha}}\,F_{\mu\nu} - \mathrm{i}\,\eta^{\beta B}\,(\sigma^{\mu})_{\beta\dot{\alpha}}\,(\bar{\Sigma}^{a})_{BA}\,\partial_{\mu}\varphi_{a}$$

$$\delta \varphi^{a} = -\mathrm{i}\,\bar{\xi}_{\dot{\alpha}A}\,(\Sigma^{a})^{AB}\,\bar{\Lambda}^{\dot{\alpha}}{}_{B} + \mathrm{i}\,\eta^{\alpha A}\,(\bar{\Sigma}^{a})_{AB}\,\Lambda_{\alpha}{}^{B}$$

- These transf. can also be obtained from the vertex operator for the gaugino $\Lambda^{\alpha A}$ and $q^{\dot\alpha A} \equiv Q^{\dot\alpha A} - \widetilde{Q}^{\dot\alpha A}$, both in the $(-1/2)$ picture

$$\left[\bar\xi_{\dot\alpha A} q^{\dot\alpha A}, V_\Lambda^{(-1/2)}(z)\right] = \bar\xi_{\dot\alpha A} \oint_z \frac{dy}{2\pi\mathrm{i}}\, j^{\dot\alpha A}(y) V_\Lambda^{(-1/2)}(z)$$

$$= -\bar\xi_{\dot\alpha A}\Lambda^{\beta B} \oint_z \frac{dy}{2\pi\mathrm{i}} \left(S^{\dot\alpha}(y) S^A(y) \mathrm{e}^{-\frac{1}{2}\phi(y)}\right)$$

$$\left(S_\beta(z) S_B(z) \mathrm{e}^{-\frac{1}{2}\phi(z)} \mathrm{e}^{\mathrm{i}p_\nu X^\nu(z)}\right) = \left(-\mathrm{i}\,\bar\xi_{\dot\alpha A} (\bar\sigma^\mu)^{\dot\alpha}{}_\beta \Lambda^{\beta A}\right)$$

$$\frac{1}{\sqrt{2}} \psi_\mu(z) \mathrm{e}^{-\phi(z)} \mathrm{e}^{\mathrm{i}p_\nu X^\nu(z)}$$

- Finally $\delta_{\bar\xi} A^\mu = \mathrm{i}\,\bar\xi_{\dot\alpha A}\,(\bar\sigma^\mu)^{\dot\alpha\beta}\,\Lambda_\beta{}^A$ or schematically $\left[\bar\xi\, q, V_\Lambda\right] = V_{\delta_{\bar\xi} A}$
- For N coincident D3 branes we add $N \times N$ Chan-Paton factors
- (-1)/(-1) strings: there are no Neumann directions so there is no momentum. These are the moduli.

  NS 4 a^μ (corresponding to the longitudinal directions of the D3 branes) and 6 χ^a (corresponding to the transverse directions to the D3's)

  R 16 fermionic moduli $M^{\alpha A}$ and $\lambda_{\dot\alpha A}$

- Vertex operators

$$
\begin{aligned}
V_a^{(-1)}(z) &= \frac{a^\mu}{\sqrt{2}}\,\psi_\mu(z)\,\mathrm{e}^{-\phi(z)} \\
V_\chi^{(-1)}(z) &= \frac{\chi^a}{\sqrt{2}}\,\psi_a(z)\,\mathrm{e}^{-\phi(z)} \\
V_M^{(-1/2)}(z) &= M^{\alpha A}\,S_\alpha(z)\,S_A(z)\,\mathrm{e}^{-\frac{1}{2}\phi(z)} \\
V_\lambda^{(-1/2)}(z) &= \lambda_{\dot\alpha A}\,S^{\dot\alpha}(z)\,S^A(z)\,\mathrm{e}^{-\frac{1}{2}\phi(z)}
\end{aligned}
$$

- The SUSY transf. that preserve also the D3 boundary are

$$\delta_{\bar\xi}\, a^\mu \;=\; \mathrm{i}\,\bar\xi_{\dot\alpha A}\,(\bar\sigma^\mu)^{\dot\alpha\beta}\, M_\beta{}^A$$
$$\delta_{\bar\xi}\,\chi^a \;=\; -\,\mathrm{i}\,\bar\xi_{\dot\alpha A}\,(\Sigma^a)^{AB}\,\lambda^{\dot\alpha}{}_B$$
$$\delta_{\bar\xi}\, M^{\alpha A} \;=\; 0 \quad , \quad \delta_{\bar\xi}\,\lambda_{\dot\alpha A} = 0$$

- A superposition of k D(-1) branes gives $k \times k$ Chan-Paton factors and

$$\delta_{\bar\xi}\, M^{\alpha A} \;=\; -\,\bar\xi_{\dot\beta B}\,(\bar\sigma^\mu)^{\dot\beta\alpha}\,(\Sigma^a)^{BA}\,[\chi_a, a_\mu]$$
$$\delta_{\bar\xi}\,\lambda_{\dot\alpha A} \;=\; \frac{1}{2}\,\bar\xi_{\dot\alpha B}\,(\bar\Sigma^{ab})^B{}_A\,[\chi_a,\chi_b] + \frac{1}{2}\,\bar\xi_{\dot\beta A}\,(\bar\sigma^{\mu\nu})^{\dot\beta}{}_{\dot\alpha}\,[a_\mu, a_\nu]$$

- To derive these transf. from the vertex formalism, we need auxiliary fields
- 3/(-1) and (-1)/3 strings: 4 directions (longitudinal to the D3 brane) have mixed b.c.

NS the ψ^μ have integer-moded expansions. The massless states are two bosonic Weyl spinors w and $\bar{w}$

R ψ^μ have half-integer mode expansions. The massless states form two fermionic Weyl spinors μ and $\bar{\mu}$

- The vertex operators (in the (-1) and $(-1/2)$ superghost picture) are

$$\begin{aligned} V_{w}^{(-1)}(z) &= w_{\dot\alpha}\,\Delta(z)\,S^{\dot\alpha}(z)\,\mathrm{e}^{-\phi(z)} \\ V_{\bar w}^{(-1)}(z) &= \bar w_{\dot\alpha}\,\bar\Delta(z)\,S^{\dot\alpha}(z)\,\mathrm{e}^{-\phi(z)} \\ V_{\mu}^{(-1/2)}(z) &= \mu^{A}\,\Delta(z)\,S_{A}(z)\,\mathrm{e}^{-\frac{1}{2}\phi(z)} \\ V_{\bar\mu}^{(-1/2)}(z) &= \bar\mu^{A}\,\bar\Delta(z)\,S_{A}(z)\,\mathrm{e}^{-\frac{1}{2}\phi(z)} \end{aligned}$$

- The unbroken SUSY give one linear and one non-linear transf.

$$\begin{aligned} \delta_{\bar\xi}\,w_{\dot\alpha} &= -\,\mathrm{i}\,\bar\xi_{\dot\alpha A}\,\mu^{A} \\ \delta_{\bar\xi}\,\mu^{A} &= -\,\frac{1}{\sqrt{2}}\,\bar\xi_{\dot\alpha B}\,(\Sigma^{a})^{BA}\,w^{\dot\alpha}\,\chi_{a} \end{aligned}$$

Effective actions

- I now compute the tree level string amplitude and do the the field theory limit $\alpha' \to 0$. I start with the 3/3 strings

$$\mathcal{A}_{(\bar{\Lambda} A \Lambda)} = \left\langle\!\!\left\langle V_{\bar{\Lambda}}^{(-1/2)} \, V_{A}^{(-1)} \, V_{\Lambda}^{(-1/2)} \right\rangle\!\!\right\rangle$$

$$\equiv \quad C_4 \int \frac{\prod_i dz_i}{dV_{123}} \, \left\langle V_{\bar{\Lambda}}^{(-1/2)}(z_1) \, V_{A}^{(-1)}(z_2) \, V_{\Lambda}^{(-1/2)}(z_3) \right\rangle$$

$$= \quad -\frac{2\,\mathrm{i}}{g_{\mathrm{YM}}^2} \,\mathrm{Tr}\left(\bar{\Lambda}_{\dot{\alpha} A} \, \bar{\not{A}}^{\dot{\alpha}\beta} \, \Lambda_{\beta}{}^{A} \right)$$

- Repeating the computation for the other possible couplings leads to

$$\mathcal{S}_{\rm SYM} = \frac{1}{g^2_{\rm YM}} \int d^4x \ {\rm Tr}\Big\{ \frac{1}{2} F^{\,2}_{\mu\nu} - \ 2\,\bar\Lambda_{\dot\alpha A}\,\bar{\not{\mathcal{D}}}^{\dot\alpha\beta}\,\Lambda_\beta^{\ A}$$

$$+ \ (\mathcal{D}_\mu\varphi_a)^2 \ - \ \frac{1}{2}\ [\varphi_a, \varphi_b]^2 - {\rm i}\,(\Sigma^a)^{AB}\,\bar\Lambda_{\dot\alpha A}\big[\varphi_a, \bar\Lambda^{\dot\alpha}{}_B\big]$$

$$- \,{\rm i}\,(\bar\Sigma^a)_{AB}\,\Lambda^{\alpha A}\Big[\varphi_a, \Lambda_\alpha^{\ B}\Big] \ \Big\}$$

- In the (-1)/(-1) case $\mathcal{A}_{(\lambda a M)} = \left\langle\!\left\langle V_\lambda^{(-1/2)}\, V_a^{(-1)}\, V_M^{(-1/2)} \right\rangle\!\right\rangle$ leads to $-\frac{\mathrm{i}}{g_0^2}\,\mathrm{tr}\left(\lambda_{\dot\alpha A}\,\left[\not{a}^{\dot\alpha\beta}\,,\,M_\beta{}^A\right]\right)$ and to an action $\mathcal{S}_{(-1)} = \mathcal{S}_{\rm c} + \mathcal{S}_{\rm q}$ with

$$\mathcal{S}_{\rm c} = \frac{\mathrm{i}}{g_0^2}\,\mathrm{tr}\Big\{\lambda_{\dot\alpha A}\left[\not{a}^{\dot\alpha\beta}, M_\beta{}^A\right] - \frac{1}{2}\,(\Sigma^a)^{AB}\,\lambda_{\dot\alpha A}\left[\chi_a, \lambda^{\dot\alpha}{}_B\right]$$

$$-\frac{1}{2}\,(\bar\Sigma^a)_{AB}\,M^{\alpha A}\left[\chi_a, M_\alpha{}^B\right]\Big\}$$

$$\mathcal{S}_{\rm q} = -\frac{1}{g_0^2}\mathrm{tr}\Big\{\frac{1}{4}\left[a_\mu, a_\nu\right]^2 + \frac{1}{2}\left[a_\mu, \chi_a\right]^2 + \frac{1}{4}\left[\chi_a, \chi_b\right]^2\Big\}$$

- In turn $\mathcal{S}_{\rm q}$ can be linearized, leading to

$$\mathcal{S}' = \frac{1}{g_0^2}\mathrm{tr}\Big\{\frac{1}{2}D_c^{\,2} + \frac{1}{2}\,D_c\bar{\eta}^c_{\mu\nu}\,[a^\mu, a^\nu] + \frac{1}{2}\,Y_{\mu a}^{\,2} + Y_{\mu a}\,[a^\mu, \chi^a] + \frac{1}{4}Z_{ab}^{\,2} + \frac{1}{2}\,Z_{ab}\left[\chi^a, \chi^b\right]\Big\}$$

- and the vertices

$$V_D^{(0)}(z) = \frac{1}{2}\,D_c\,\bar{\eta}^c_{\mu\nu}\,\psi^\nu(z)\psi^\mu(z)$$

$$V_Y^{(0)}(z) = Y_{\mu a}\,\psi^a(z)\psi^\mu(z)$$

$$V_Z^{(0)}(z) = \frac{1}{2}\,Z_{ab}\,\psi^b(z)\psi^a(z)$$

- Now $[\bar{\xi}\, q, V_D] = V_{\delta_{\bar{\xi}}\,\lambda}$ leads to
  $\delta_{\bar{\xi}}\lambda_{\dot{\alpha}A} = -\frac{1}{4}\,\bar{\xi}_{\dot{\beta}A}(\bar{\sigma}^{\mu\nu})^{\dot{\beta}}{}_{\dot{\alpha}} D_c\,\bar{\eta}^c_{\mu\nu}$
- Finally the (-1)/3 and 3/(-1) parts lead to

$$\mathcal{S}'' = \frac{2\mathrm{i}}{g_0^2}\,\mathrm{tr}\Big\{\Big(\bar{\mu}^A{}_u w^u_{\dot{\alpha}} + \bar{w}_{\dot{\alpha}u}\mu^{Au}\Big)\lambda^{\dot{\alpha}}_A - D_c W^c$$

$$+\frac{1}{2}(\bar{\Sigma}^a)_{AB}\,\bar{\mu}^A{}_u\mu^{Bu}\chi_a - \mathrm{i}\chi_a\bar{w}_{\dot{\alpha}u}w^{\dot{\alpha}u}\chi^a\Big\}$$

- Finally the moduli action is

$$S = \mathrm{tr}\Big\{$$

$$Y'^{2}_{\mu a} + 2Y'_{\mu a}\,\left[a'^{\mu}, \chi'^{a}\right] + \frac{1}{4} Z'^{2}_{ab} + \chi'_{a}\,\bar{w}'_{\dot\alpha u}\,w'^{\dot\alpha u}\chi'^{a} +$$

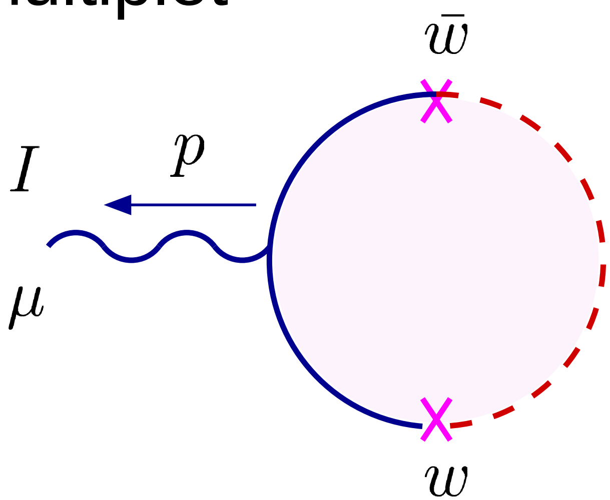
$$\frac{\mathrm{i}}{2}\,(\bar\Sigma^{a})_{AB}\bar{\mu}'^{A}{}_{u}\,\mu'^{Bu}\,\chi'_{a} - \frac{\mathrm{i}}{4}\,(\bar\Sigma^{a})_{AB}\,M'^{\alpha A}\left[\chi'_{a}, M'^{B}_{\alpha}\right]$$

$$+\mathrm{i}\left(\bar{\mu}'^{A}{}_{u} w'_{\dot\alpha}{}^{u} + \bar{w}'_{\dot\alpha u}\mu'^{Au} + \left[M'^{\beta A}, a'_{\beta\dot\alpha}\right]\right)\lambda'^{\dot\alpha}{}_{A}$$

$$-\mathrm{i}D'_{c}\Big(W'^{c} + \mathrm{i}\bar\eta^{c}_{\mu\nu}\,\left[a'^{\mu}, a'^{\nu}\right]\Big)\Big\}$$

The instanton from mixed disks

- D(-1) branes are sources for the fields in the gauge supermultiplet



- In momentum space

$$A^I_\mu(p;\bar{w},w) = \left\langle\!\!\left\langle V^{(-1)}_{\bar{w}} \mathcal{V}^{(0)}_{A^I_\mu}(-p)\, V^{(-1)}_{w} \right\rangle\!\!\right\rangle =$$

$$\mathrm{i}(T^I)^v{}_u p^\nu \bar{\eta}^c_{\nu\mu} \left(w_{\dot{\alpha}}{}^u (\tau_c)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{w}^{\dot{\beta}}{}_v \right) \mathrm{e}^{-\mathrm{i}p\cdot x_0}$$

- Attaching the gluon propagator $\delta_{\mu\nu}/p^2$ and FT

$$
\begin{aligned}
A^I_\mu(x) &= \int \frac{d^4p}{(2\pi)^2}\, A^I_\mu(p;\bar{w},w)\,\frac{1}{p^2}\,\mathrm{e}^{\mathrm{i}p\cdot x} \\
&= -2\,(T^I)^v{}_u\,\left(w_{\dot\alpha}{}^u\,(\tau_c)^{\dot\alpha}{}_{\dot\beta}\,\bar{w}^{\dot\beta}{}_v\right)\,\bar\eta^c_{\nu\mu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4} \\
&= 4\rho^2\mathrm{Tr}(T^I\,t_c)\bar\eta^c_{\mu\nu}\frac{(x-x_0)^\nu}{(x-x_0)^4}
\end{aligned}
$$

- Analogous formulae hold for the other members of the supermultiplet

ADHM construction

- The basic objects in the ADHM construction are $[N+2k]\times[2k]$ matrices $\Delta(x) = a + bx$

$$a \equiv \begin{pmatrix} w_{\dot\alpha}{}^{ui} \\ a'_{\alpha\dot\beta\ li} \end{pmatrix} \quad , \quad b = \begin{pmatrix} 0 \\ \mathbf{1}_{[2k]\times[2k]} \end{pmatrix}$$

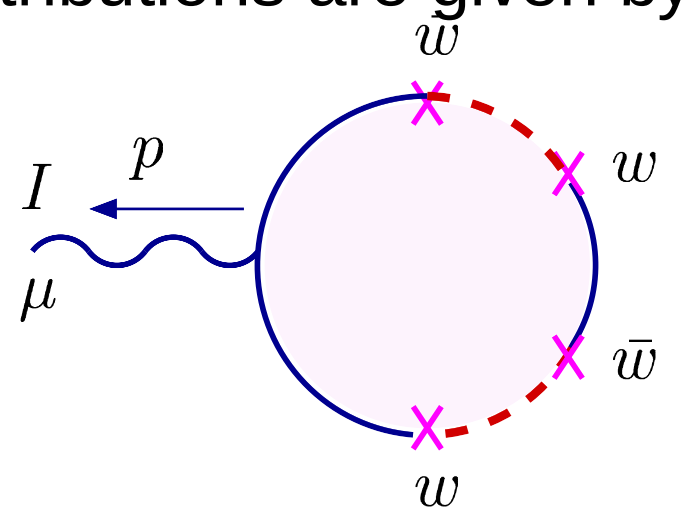
- The constraints are $\bar\Delta\,\Delta = f^{-1}_{k\times k}\mathbf{1}_{[2]\times[2]}$ which for $k=1$ give $\bar w^{\dot\alpha}{}_u w_{\dot\beta}{}^u = \rho^2\delta^{\dot\alpha}_{\dot\beta}$ whose solution is

$$||w_{\dot\alpha}{}^u|| = \left|\left|\bar w^{\dot\alpha}{}_u\right|\right| = \rho T \begin{pmatrix} 0_{[N-2]\times[2]} \\ 1_{[2]\times[2]} \end{pmatrix}$$

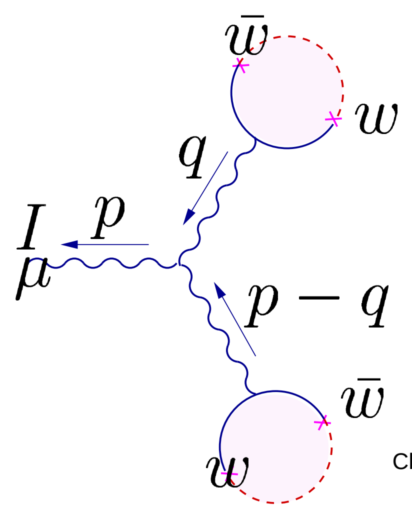
$$T \in \mathrm{SU}(N)/\mathrm{SU}(N-2)$$

- The gauge field is $(\widehat{A}_\mu)^u{}_v = \frac{\rho^2}{x^2(x^2+\rho^2)}(\bar\sigma_{\nu\mu})^u{}_v\, x^\nu$ with $(\bar\sigma_{\nu\mu})_u{}^v = \begin{pmatrix} 0_{[N-2]\times[N-2]} & 0_{[N-2]\times[2]} \\ 0_{[2]\times[N-2]} & (\bar\sigma_{\nu\mu})_{\dot\alpha}{}^{\dot\beta} \end{pmatrix}$.
- For $SU(N)$ we get $\widehat{A}_\mu = T\,\widehat{A}_\mu\,T^{-1}$

- Higher order contributions are given by



- In the limit $\alpha' \to 0$



- Recovering the expansion

$$A_\mu^c(x) = 2\rho^2\,\bar\eta^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^2\Big[(x-x_0)^2+\rho^2\Big]}$$

$$\simeq 2\rho^2\,\bar\eta^c_{\mu\nu}\,\frac{(x-x_0)^\nu}{(x-x_0)^4}\left(1-\frac{\rho^2}{(x-x_0)^2}+\ldots\right)$$

Instanton calculus

- the tree-level scattering amplitude among n states of the 3/3 strings

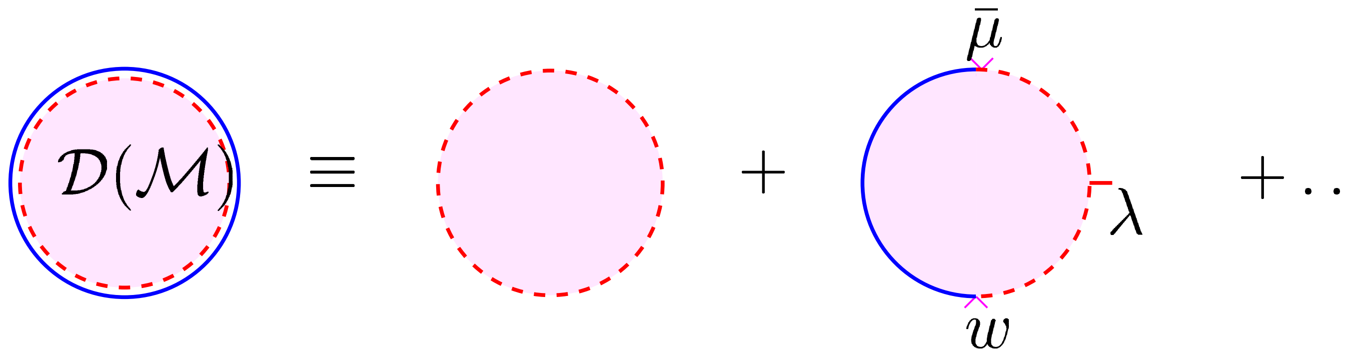
$$\mathcal{A}_{\phi_1..\phi_n} = \phi_n(p_n)..\phi_1(p_1)\left\langle\!\!\left\langle \mathcal{V}_{\phi_1}(p_1)..\mathcal{V}_{\phi_n}(p_n)\right\rangle\!\!\right\rangle$$

- in the limit $\alpha' \to 0$ and extracting the 1PI part

$$-\int \frac{d^4p_1}{(2\pi)^2}..\frac{d^4p_n}{(2\pi)^2}\,\phi_n(p_n)..\phi_1(p_1)\,\left\langle\!\!\left\langle \mathcal{V}_{\phi_1}(p_1)..\mathcal{V}_{\phi_n}(p_n)\right\rangle\!\!\right\rangle\Big|^{\mathrm{1PI}}_{\alpha'\to}$$

- In the presence of D(-1) branes we get a contribution from world sheets with a part of their boundary on the D-instantons

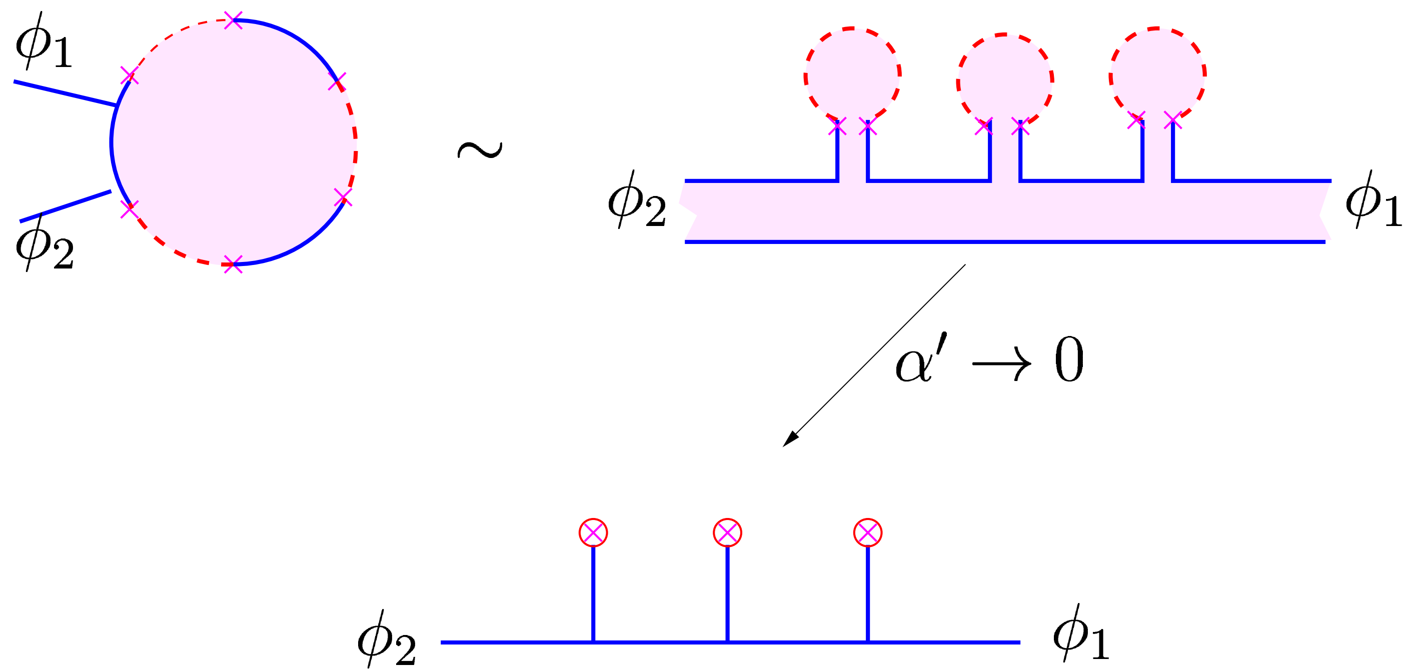
- $\mathcal{D}(\mathcal{M})$ is the sum of all disks with all possible insertions of the moduli $\mathcal{M}$ of the k instantons



- The vacuum contribution of the “disk” $\mathcal{D}(\mathcal{M})$ is

$$\langle\!\langle\, 1 \,\rangle\!\rangle_{\mathcal{D}(\mathcal{M})} \overset{\alpha'\to 0}{\simeq} -\, S[\mathcal{M}] \equiv -\frac{8\pi^2 k}{g_{\rm YM}^2} - S_{\rm moduli}$$

- The integration over $\mathcal{M}$ is the analogue of what one does in quantum field theory, where the path integral describing a correlator is split into different topological sectors

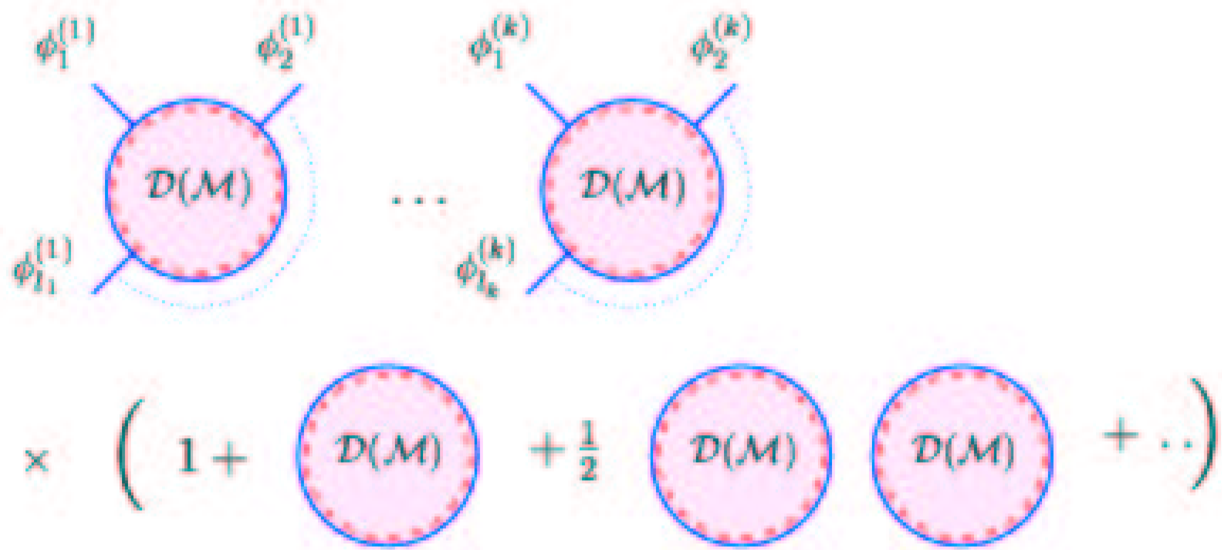


- The integration over the moduli $\mathcal{M}$ has consequences. Correlators disconnected from the 2-dim viewpoint are connected for the 4-dim theory on the D3 branes

i) $\frac{1}{\ell!}\left\langle\!\!\left\langle \mathcal{V}_{\phi_1}(p_1)\ldots\mathcal{V}_{\phi_n}(p_n)\right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})}\left(\langle\!\langle\, 1\,\rangle\!\rangle_{\mathcal{D}(\mathcal{M})}\right)^{\ell}$

ii) $\left\langle\!\!\left\langle \mathcal{V}_{\phi_1}(p_1)\,\mathcal{V}_{\phi_2}(p_2)\right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})}\left\langle\!\!\left\langle \mathcal{V}_{\phi_3}(p_3)\ldots\mathcal{V}_{\phi_n}(p_n)\right\rangle\!\!\right\rangle_{\mathcal{D}(\mathcal{M})}$
$\mathrm{e}^{\langle\!\langle\, 1\,\rangle\!\rangle_{\mathcal{D}(\mathcal{M})}}$

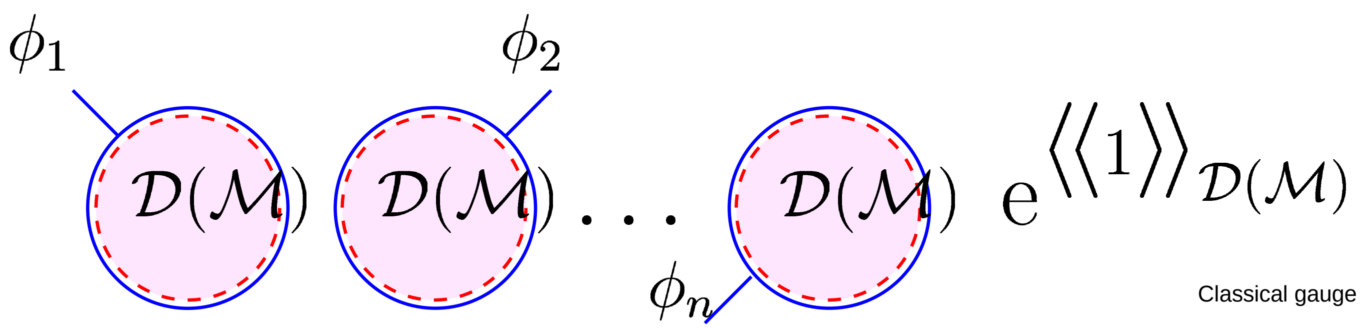
- Pictorically



- Since each expectation value on $\mathcal{D}(\mathcal{M})$ is proportional to $C_0 \propto g_s^{-1}$ the dominant contribution for small g_s is the one in which a single vertex $\mathcal{V}_\phi$ is inserted in each disk

$$\Big\langle \phi_1(p_1) \ldots \phi_n(p_n) \Big\rangle \Big|^{\mathrm{D-inst.}}_{\mathrm{amput.}} =$$

$$\int d\mathcal{M} \, \Big\langle\!\Big\langle \mathcal{V}_{\phi_1}(-p_1) \Big\rangle\!\Big\rangle_{\mathcal{D}(\mathcal{M})} .. \Big\langle\!\Big\langle \mathcal{V}_{\phi_n}(-p_n) \Big\rangle\!\Big\rangle_{\mathcal{D}(\mathcal{M})} \, \mathrm{e}^{\langle\!\langle 1 \rangle\!\rangle_{\mathcal{D}(\mathcal{M})}}$$



- Remembering that

$\phi^{\rm disk}(x;\mathcal{M}) = \int \frac{d^4p}{(2\pi)^2}\, {\rm e}^{{\rm i}p\cdot x}\, \frac{1}{p^2}\, \Big\langle\!\!\Big\langle \mathcal{V}_\phi(-p) \Big\rangle\!\!\Big\rangle_{\mathcal{D}(\mathcal{M})}\Big|_{\alpha'\to 0}$ and

$\phi(x;\mathcal{M})^{\rm disk} = \phi^{\rm cl}(x;\mathcal{M})$

- We compare with the field theory prescription

$$\Big\langle \phi_1(x_1)\ldots\phi_n(x_n)\Big\rangle\Big|_{\rm inst.} =$$

$$\int d\mathcal{M}\; \phi_1^{\rm cl}(x_1;\mathcal{M})\cdots\; \phi_n^{\rm cl}(x_n;\mathcal{M})\,{\rm e}^{-S[\mathcal{M}]}$$