

# Generalisation of the Yang-Mills Theory

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## *Extension of Poincaré Algebra*

## *Extension of Yang-Mills Theory*

*The Lagrangian*

*Tree and 1-loop amplitudes*

*Asymptotic Freedom of Tensorgluons*

## *Proton Structure, its Spin and Tensorgluons*

*Proton Spin*

*Grand Unification*

- 1.G.Savvidy, Generalization of Yang-Mills theory  
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- 2.G.Savvidy, Extension of Poincaré Group and Tensorgluons  
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- 3.I.Antoniadis, L. Brink, G. Savvidy  
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EXTENSION OF THE ALGEBRA OF POINCARÉ GROUP GENERATORS AND VIOLATION OF P INVARIANCE

Yu.A. Gol'fand and E.P. Likhtman  
 Physics Institute, USSR Academy of Sciences  
 Submitted 10 March 1971  
 ZhETF Pis. Red. 13, No. 8, 452 - 455 (20 April 1971)

One of the main requirements imposed on quantum field theory is invariance of the theory to the Poincaré group [1]. However, only a fraction of the interactions satisfying this requirement is realized in nature. It is possible that these interactions, unlike others, have a higher degree of symmetry. It is therefore of interest to study different algebras and groups, the invariance with respect to which imposes limitations on the form of the elementary particle interaction. In the present paper we propose, in constructing the Hamiltonian formulation of the quantum field theory, to use as the basis a special algebra  $\mathcal{K}$ , which is an extension of the algebra  $\mathcal{P}$  of the Poincaré group generators. The purpose of the paper is to find such a realization of the algebra  $\mathcal{K}$ , in which the Hamiltonian operator describes the interaction of quantized fields.

The extension of the algebra  $\mathcal{P}$  is carried out in the following manner: we add to the generators  $P_\mu$  and  $M_{\mu\nu}$  the bispinor generators  $\bar{W}_\alpha$  and  $\bar{W}_\beta$ , which we shall call the generators of spinor translations. In order to obtain the algebra  $\mathcal{K}$ , it is necessary to find the Lorentz-invariant form of the permutation relations between the translation generators. In order not to violate subsequently the connection between the spin and statistics, we shall consider anticommutators of the operators  $\bar{W}_\alpha$  and  $\bar{W}_\beta$ . A generalization of the Jacobi identities imposes stringent limitations on the form of the possible commutation relations between the algebra operators. We confine ourselves to consideration of only those algebras  $\mathcal{K}$ , in which there are no subalgebras  $\mathcal{Q}$  such that  $\mathcal{P} \subset \mathcal{Q}$  and  $\mathcal{P} \neq \mathcal{Q}$ . This choice is governed by the fact that the remaining algebras  $\mathcal{K}$  are obtained by further extending the algebras  $\mathcal{K}$ , and the field theories corresponding to them will have a still higher degree of symmetry.

An investigation of the algebras  $\mathcal{K}$  has shown that upon spatial inversion they do not go over into themselves for any choice of the structure constants of the algebra. As a result, in a field theory that is invariant against such an algebra, the parity should not be conserved, and the form of the nonconservation is completely determined by the algebra itself. We shall stop to discuss one of the algebras  $\mathcal{K}$ :

$$[M_{\mu\nu}, M_{\sigma\lambda}]_- = i(\delta_{\mu\sigma}M_{\nu\lambda} + \delta_{\nu\lambda}M_{\mu\sigma} - \delta_{\mu\lambda}M_{\nu\sigma} - \delta_{\nu\sigma}M_{\mu\lambda}); [P_\mu, P_\nu]_- = 0; \quad (1a)$$

$$[M_{\mu\nu}, P_\lambda]_- = i(\delta_{\mu\lambda}P_\nu - \delta_{\nu\lambda}P_\mu); [M_{\mu\nu}, W]_- = \frac{i}{4} [\gamma_\mu, \gamma_\nu] W; \bar{W} = W^* \gamma_0.$$

$$[W_\alpha, \bar{W}_\beta]_- = \gamma_\alpha \beta; [W, W]_- = 0; [P_\mu, W]_- = 0, \quad (1b)$$

We add to the generators  $P_\mu$  and  $M_{\mu\nu}$  the new generators  $L_a^{\lambda_1 \dots \lambda_s}$  symmetric over Lorentz indices  $\lambda_1 \dots \lambda_s$  and  $s=0,1,2,\dots$

The extension of the Poincaré algebra  $L_G(\mathcal{P})$  is:

$$[P^\mu, P^\nu] = 0,$$

$$[M^{\mu\nu}, P^\lambda] = i(\eta^{\lambda\nu} P^\mu - \eta^{\lambda\mu} P^\nu),$$

$$[M^{\mu\nu}, M^{\lambda\rho}] = i(\eta^{\mu\rho} M^{\nu\lambda} - \eta^{\mu\lambda} M^{\nu\rho} + \eta^{\nu\lambda} M^{\mu\rho} - \eta^{\nu\rho} M^{\mu\lambda}),$$

$$[P^\mu, L_a^{\lambda_1 \dots \lambda_s}] = 0,$$

$$[M^{\mu\nu}, L_a^{\lambda_1 \dots \lambda_s}] = i(\eta^{\lambda_1\nu} L_a^{\mu\lambda_2 \dots \lambda_s} + \dots + -\eta^{\lambda_s\mu} L_a^{\lambda_1 \dots \lambda_{s-1}\nu}),$$

$$[L_a^{\lambda_1 \dots \lambda_n}, L_b^{\lambda_{n+1} \dots \lambda_s}] = i f_{abc} L_c^{\lambda_1 \dots \lambda_s} \quad (s = 0, 1, 2, \dots).$$

The generators  $L_a^{\lambda_1 \dots \lambda_s}$  carry *internal charges*  $a$  and *high spins*.

The algebra  $L_G(\mathcal{P})$  is gauge invariant:

$$L_a^{\lambda_1 \dots \lambda_s} \rightarrow L_a^{\lambda_1 \dots \lambda_s} + \\ + \sum_1 P^{\lambda_1} L_a^{\lambda_2 \dots \lambda_s} + \sum_2 P^{\lambda_1} P^{\lambda_2} L_a^{\lambda_3 \dots \lambda_s} + \dots + P^{\lambda_1} \dots P^{\lambda_s} L_a,$$

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- ▶ This is “off-shell” symmetry, the operator  $P^2$  has any value.
- ▶ These generators are “gauge generators”.
- ▶ *All representations of the  $L_a^{\lambda_1 \dots \lambda_s}$ ,  $s = 1, 2, \dots$  are defined modulo longitudinal terms.*

The reducible representation of  $L_G(\mathcal{P})$  has the following form:

$$\begin{aligned}P^\mu &= k^\mu, \\M^{\mu\nu} &= i(k^\mu \frac{\partial}{\partial k_\nu} - k^\nu \frac{\partial}{\partial k_\mu}) + i(\xi^\mu \frac{\partial}{\partial \xi_\nu} - \xi^\nu \frac{\partial}{\partial \xi_\mu}), \\L_a^{\lambda_1 \dots \lambda_s} &= \xi^{\lambda_1} \dots \xi^{\lambda_s} \otimes L_a,\end{aligned}$$

The gauge transformation of  $L_a^{\lambda_1 \dots \lambda_s}$  induces the transformation

$$\xi^\mu \rightarrow \xi^\mu + \alpha k^\mu,$$

which is a gauge transformation of the photon polarisation vector. The generators  $L_a^{\lambda_1 \dots \lambda_s}$  are indeed gauge generators.

The irreducible transversal representation of the generators  $L_a^{\lambda_1 \dots \lambda_s}$

$$L_a^{\lambda_1 \dots \lambda_s} = \prod_{n=1}^s (\xi k^{\lambda_n} + e^{i\varphi} e_+^{\lambda_n} + e^{-i\varphi} e_-^{\lambda_n}) \oplus L_a,$$

where the helicity vectors are  $e_{\pm}^{\lambda} = (e_1^{\lambda} \mp i e_2^{\lambda})/2$ ,

$L_a \in SU(N)$

$k_{\mu}$  is the momentum vector  $k \cdot e_i = 0$

the  $\xi$  and  $\varphi$  are independent variables on the cylinder

$$\varphi \in S^1, \xi \in R^1.$$

Using transversal representation one can calculate Killing forms:

$$\begin{aligned}
 L_G : \quad \langle L_a; L_b \rangle &= \delta_{ab}, & L_{\mathcal{P}} : \quad \langle P^\mu; P^\nu \rangle &= 0 \\
 & & & \langle M_{\mu\nu}; P_\lambda \rangle = 0 \\
 & & & \langle M^{\mu\nu}; M^{\lambda\rho} \rangle = \eta^{\mu\lambda}\eta^{\nu\rho} - \eta^{\mu\rho}\eta^{\nu\lambda} \\
 & & & \langle P^\mu; L_a^{\lambda_1 \dots \lambda_s} \rangle = 0, \\
 & & & \langle M^{\mu\nu}; L_a^{\lambda_1 \dots \lambda_s} \rangle = 0,
 \end{aligned}$$

$L_G(\mathcal{P}) :$

$$\begin{aligned}
 \langle L_a^{\lambda_1 \dots \lambda_n}; L_b^{\lambda_{n+1} \dots \lambda_{2s+1}} \rangle &= 0, & s &= 0, 1, 2, 3, \dots \\
 \langle L_a^{\lambda_1 \dots \lambda_n}; L_b^{\lambda_{n+1} \dots \lambda_{2s}} \rangle &= \delta_{ab} s! (\eta^{\lambda_1 \lambda_2} \eta^{\lambda_3 \lambda_4} \dots \eta^{\lambda_{2s-1} \lambda_{2s}} + \text{per})
 \end{aligned}$$

where  $L_G$  - is internal algebra,  $L_{\mathcal{P}}$  - is Poincaré algebra,

$L_G(\mathcal{P})$ - the unifying algebra

The Yang-Mills non-Abelian tensor gauge fields

$$\mathcal{A}_\mu(x, L) = \sum_{s=0}^{\infty} \frac{1}{s!} A_{\mu\lambda_1\dots\lambda_s}^a(x) L_a^{\lambda_1\dots\lambda_s}. \quad (1)$$

The field strength tensor

$$\mathcal{G}_{\mu\nu}(x, e) = \partial_\mu \mathcal{A}_\nu(x, e) - \partial_\nu \mathcal{A}_\mu(x, e) - ig[\mathcal{A}_\mu(x, e) \mathcal{A}_\nu(x, e)] \quad (2)$$

and the Lagrangian density is

$$\mathcal{L}(x) = \langle \mathcal{G}_{\mu\nu}^a(x, L) \mathcal{G}_{\mu\nu}^a(x, L) \rangle. \quad (3)$$

In components the Lagrangian is

$$\begin{aligned} \mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + &= - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a \\ &- \frac{1}{4} G_{\mu\nu,\lambda}^a G_{\mu\nu,\lambda}^a - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu,\lambda\lambda}^a + \\ &+ \frac{1}{4} G_{\mu\nu,\lambda}^a G_{\mu\lambda,\nu}^a + \frac{1}{4} G_{\mu\nu,\nu}^a G_{\mu\lambda,\lambda}^a + \frac{1}{2} G_{\mu\nu}^a G_{\mu\lambda,\nu\lambda}^a + \end{aligned}$$

where the field strength tensors are:

$$\begin{aligned} G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c, \\ G_{\mu\nu,\lambda}^a &= \partial_\mu A_{\nu\lambda}^a - \partial_\nu A_{\mu\lambda}^a + gf^{abc} ( A_\mu^b A_{\nu\lambda}^c + A_{\mu\lambda}^b A_\nu^c ), \end{aligned}$$

.....

## Helicity spectrum of the tensorgluons

$\pm 1$

$\pm 2, \quad 0$

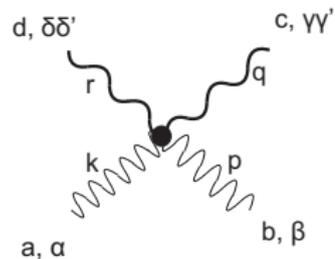
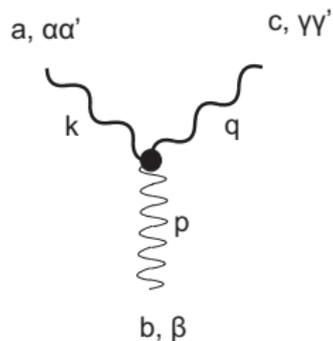
$\pm 3, \quad \pm 1, \quad \pm 1$

$\pm 4, \quad \pm 2, \quad \pm 2, \quad 0$

$\pm 5, \quad \pm 3, \quad \pm 3, \quad \pm 1, \quad \pm 1$

$\pm 6, \quad \pm 4, \quad \pm 4, \quad \pm 2, \quad \pm 2, \quad 0$

.....,



Interaction Vertices of gluons and tensorgluons

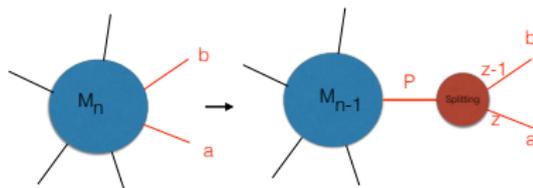
Tree level scattering amplitudes of (n-2)-gluons and 2-tensorgluons calculated using BCFW formalism. G.Georgiou and G.S. IJMP 2011.

$$\hat{M}_n(1^+, ..i^-, ...k^{+s}, ..j^{-s}, ..n^+) =$$
$$= ig^{n-2}(2\pi)^4 \delta^{(4)}(P^{ab}) \frac{\langle ij \rangle^4}{\prod_{l=1}^n \langle ll+1 \rangle} \left( \frac{\langle ij \rangle}{\langle ik \rangle} \right)^{2s-2},$$

They reduce to the Parke-Taylor formula when  $s = 1$ .

The collinear behavior:

$$M_n^{tree}(\dots, a^{\lambda_a}, b^{\lambda_b}, \dots) \xrightarrow[a \parallel b]{} \sum_{\lambda=\pm 1} Split_{-\lambda}^{tree}(a^{\lambda_a}, b^{\lambda_b}) \times M_{n-1}^{tree}(\dots, P^\lambda, \dots),$$



Antoniadis and Savvidy, Mod.Phys.Lett.(2012)

The splitting probabilities are:

$$P_{TG}(z) = C_2(G) \left[ \frac{z^{2s+1}}{(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z^{2s-1}} \right],$$
$$P_{GT}(z) = C_2(G) \left[ \frac{1}{z(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z} \right],$$
$$P_{TT}(z) = C_2(G) \left[ \frac{1}{(1-z)z^{2s-1}} + \frac{z^{2s+1}}{1-z} \right].$$

$$s=0,1,2,3,\dots$$

The quark and gluon splitting probabilities of Altarelli-Parisi:

$$\begin{aligned}P_{qq}(z) &= C_2(R) \frac{1+z^2}{1-z}, \\P_{Gq}(z) &= C_2(R) \frac{1+(1-z)^2}{z}, \\P_{qG}(z) &= T(R)[z^2+(1-z)^2], \\P_{GG}(z) &= C_2(G) \left[ \frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)} \right],\end{aligned}\tag{4}$$

where  $C_2(G) = N$ ,  $C_2(R) = \frac{N^2-1}{2N}$ ,  $T(R) = \frac{1}{2}$  for the SU(N) groups.

## Generalisation of DIGLAP evolution equations

$$\begin{aligned}\dot{q}^i(x, t) &= \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y} [q^j(y, t) P_{q^i q^j}(\frac{x}{y}) + G(y, t) P_{q^i G}(\frac{x}{y})], \\ \dot{G}(x, t) &= \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y} [q^j(y, t) P_{G q^j}(\frac{x}{y}) + G(y, t) P_{GG}(\frac{x}{y}) + T(y, t) P_{GT}(\frac{x}{y})] \\ \dot{T}(x, t) &= \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dy}{y} [G(y, t) P_{TG}(\frac{x}{y}) + T(y, t) P_{TT}(\frac{x}{y})].\end{aligned}$$

The  $\alpha(t)$  is the running coupling constant ( $\alpha = g^2/4\pi$ )

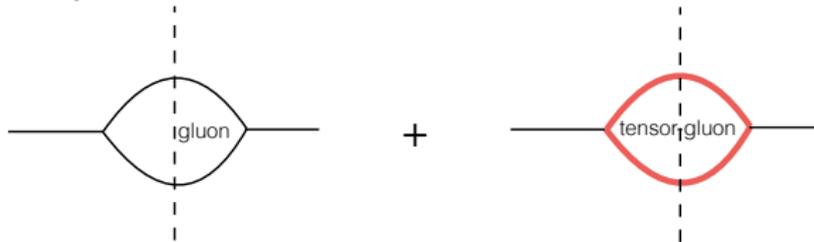
$$\alpha(t) = \frac{\alpha}{1 + b_1 \alpha t}, \quad (6)$$

where

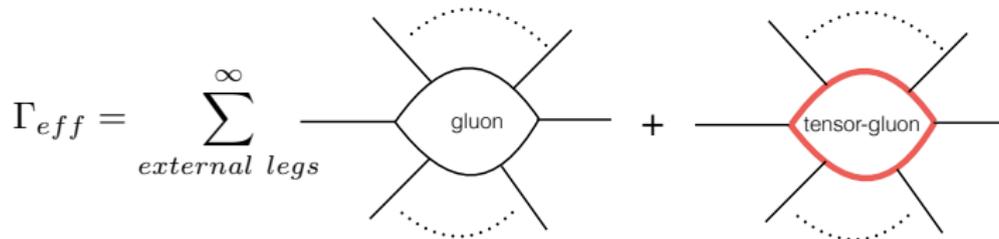
$$b_1 = b_{quarks} + b_{gluons} + b_{tensorgluons}$$

1-loop amplitudes of gluons and tensorgluons can be calculated by  
 α) unitarity or by β) one loop effective action

α) Unitarity



β) Effective Action



Both calculations gave the identical result :

$$\alpha(t) = \frac{\alpha}{1 + b_1 \alpha t} ,$$

where

$$b_1 = \frac{\sum_{s=1} (12s^2 - 1) C_2(G) - 4n_f T(R)}{12\pi} \quad s = 0, 1, 2, \dots$$

at  $s=1$  it reproduces the Gross-Wilczek-Politzer result

Tensorgluons "accelerate" the asymptotic freedom !

Generalisation of the YM effective action  $\Gamma(A)$  gives 1-loop effective action similar to G.S., Phys.Lett.B 1977

Summing the spectrum of the tensorgluons in external field

$$k_0^2 = (2n + 1 \pm 2s)gH + k_{\parallel}^2 \quad (7)$$

one can get

$$V(H) = \frac{H^2}{2} + \frac{(gH)^2}{4\pi} b_1 \left[ \ln \frac{gH}{\mu^2} - \frac{1}{2} \right], \quad (8)$$

where now

$$b_1 = \frac{12s^2 - 1}{12\pi} C_2(G), \quad (9)$$

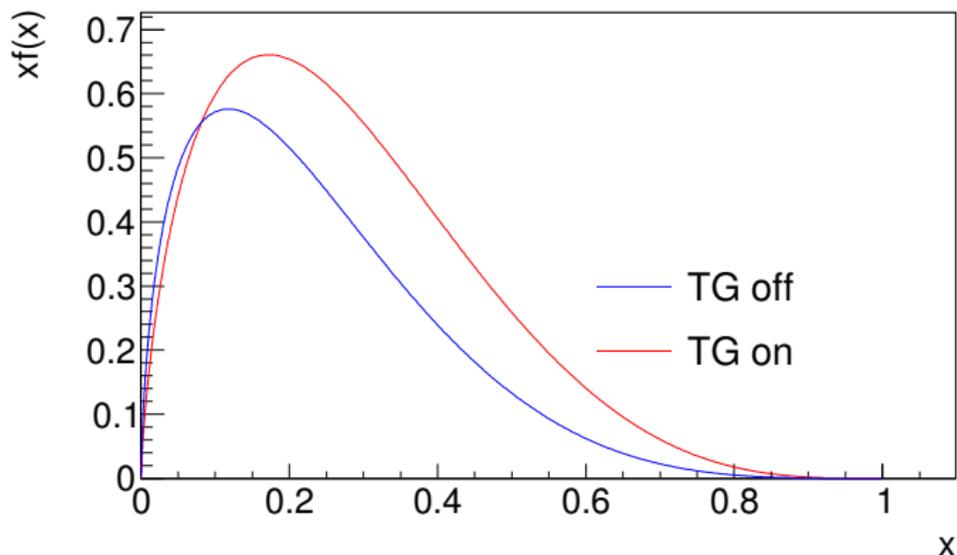


and the summation over columns gives:

$$\begin{aligned}
 b_1 &= C_2(G) \left[ \sum_{s=1}^{\infty} \frac{(12s^2 - 1)}{12\pi} + \sum_{s=0}^{\infty} \frac{(12s^2 - 1)}{12\pi} + \right. \\
 &+ \left. \sum_{s=1}^{\infty} \frac{(12s^2 - 1)}{12\pi} + \sum_{s=0}^{\infty} \frac{(12s^2 - 1)}{12\pi} + \dots \right] = \\
 &= C_2(G) \left[ \frac{1}{24\pi} - \frac{1}{12\pi} + \frac{1}{24\pi} + \dots \right] = 0,
 \end{aligned}$$

leading to the theory which is *conformally invariant* at very high energies. The above summation requires explicit regularisation and further justification.

## Example of non-singlet parton distribution function



Spin sum rule of helicity weighted distributions:

$$\frac{1}{2}\Delta\Sigma + \Delta G + \sum_s s \Delta T_s + L_z = \frac{1}{2}\hbar.$$

The lowest moment of the spin-dependent structure function

$$\Gamma_1^p = \int_0^1 dx g_1^p(x, Q^2) = I_3 + I_8 + I_0$$

The singlet part of the proton spin structure function

$$I_0 = \frac{1}{9} \left( \Delta\Sigma - n_f \frac{\alpha(Q^2)}{2\pi} \Delta G \right) \left( 1 - \frac{\alpha(Q^2)}{2\pi} \frac{3(12s^2 - 1) - 8n_f}{3(12s^2 - 1) - 2n_f} \right) +$$

$$+ n_f \frac{\alpha(Q^2)}{2\pi} \Delta T_s \frac{\sum_{k=1}^{2s+1} \frac{1}{k}}{3(12s^2 - 1) - 2n_f}.$$

How the contribution of tensor-gluons changes the high energy behavior of the coupling constants of the SM ?

The coupling constants evolve with scale as

$$\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_i(\mu)} + 2b_i \ln \frac{M}{\mu}, \quad i = 1, 2, 3, \quad (10)$$

consider only the contribution of  $s = 2$  tensor-bosons:

For the  $SU(3)_c \times SU(2)_L \times U(1)$  group with its coupling constants  $\alpha_3, \alpha_2$  and  $\alpha_1$  and six quarks  $n_f = 6$  and  $SU(5)$  unification group we will get

$$2b_3 = \frac{1}{2\pi} 54, \quad 2b_2 = \frac{1}{2\pi} \frac{10}{3}, \quad 2b_1 = -\frac{1}{2\pi} 4,$$

the solution of the system of equations (10) gives

$$\ln \frac{M}{\mu} = \frac{\pi}{58} \left( \frac{1}{\alpha_{el}(\mu)} - \frac{8}{3} \frac{1}{\alpha_s(\mu)} \right), \quad (11)$$

If one takes  $\alpha_{el}(M_Z) = 1/128$  and  $\alpha_s(M_Z) = 1/10$  one can get that coupling constants have equal strength at energies of order

$$M \sim 4 \times 10^4 GeV = 40 TeV,$$

it is much smaller than the previous GU scale  $M \sim 10^{14} GeV$   
the value of the weak angle remains intact :

$$\sin^2 \theta_W = \frac{1}{6} + \frac{5}{9} \frac{\alpha_{el}(M_Z)}{\alpha_s(M_Z)}, \quad (12)$$

the coupling constant at the unification scale is of order  
 $\bar{\alpha}(M) = 0,01$ .

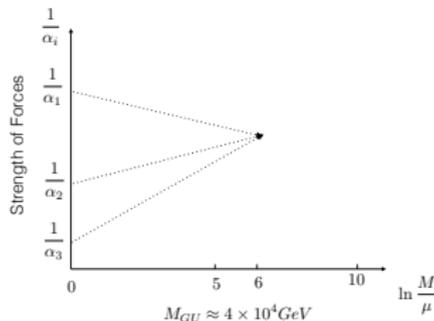
## Summary

Asymptotic Freedom of Tensorgluons of spin  $s=1,2,\dots$

$$\beta_{GYM} = -\frac{\sum_{s=1} (12s^2 - 1)C_2(G) - 4n_f T(R)}{48\pi^2} g^3$$



PROTON



$$\frac{1}{2} \Delta \Sigma + \Delta G + \sum_s s \Delta T_s + L_z = \frac{1}{2} \hbar$$

Visiting Chern Institute of Mathematics and his home in Tianjin.  
The Calendar created by Prof. Shiing-Shen Chern in which each month was devoted to an important mathematical discovery



# The Principle of Gauge Invariance and Fundamental Forces

麦克斯韦 (1831-1879) 是 19 世纪伟大的英国物理学家。1864 年，麦克斯韦导出了电磁场微分方程组。正是根据这些方程的预言，他预言了电磁波的存在。

麦克斯韦方程与爱因斯坦的相对论密切相关。以后与量子力学相结合，它也是电磁工程的基石，改变了人类的生活面貌，同时有重要数学意义。

杨振宁进一步建立了杨-米尔斯的规范场理论 (1954 年)，是麦氏理论的推广，成为现代物理理论柱石之一。

爱因斯坦的时空理论

规范场理论示意图

电子

玻色子

$$\text{div } B = 0, \quad \text{curl } E + \frac{\partial B}{\partial t} = 0,$$
$$\text{div } E = 4\pi\rho, \quad \text{curl } B - \frac{\partial E}{\partial t} = 4\pi j_0.$$

