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The Gonihedric Paradigm Extension of the Ising Model

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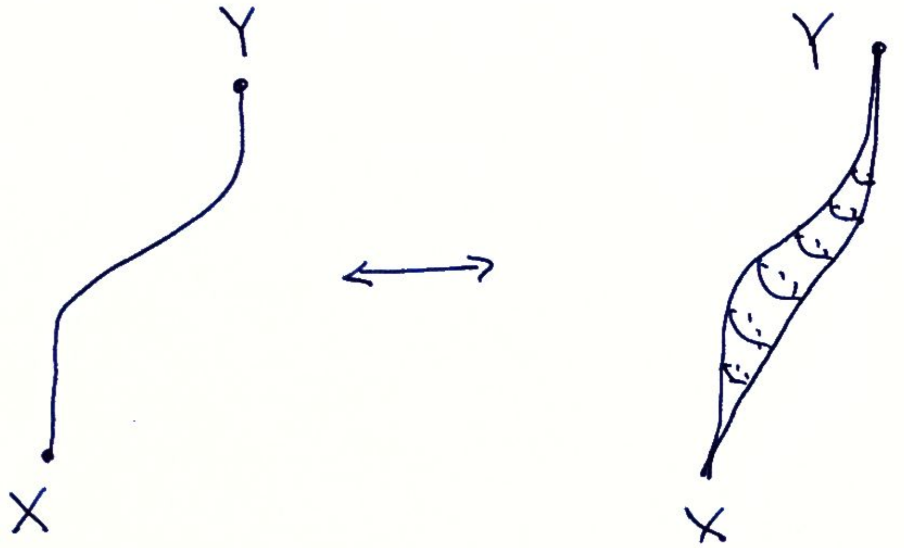
1. Extension of the Feynman path integral to surfaces.



$$\sum_{\text{paths}} e^{-\beta L_{XY}} \longleftrightarrow \sum_{\text{surfaces}} e^{-\beta L_{XY}}$$

$$[L_{XY}] = cm$$

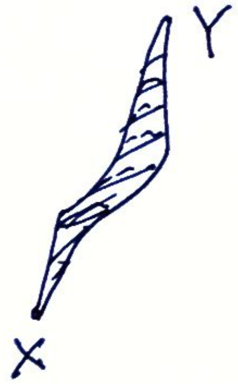
It is required that the action L_{XY} should measure the surfaces in terms of lengths.

Curve $\qquad\qquad$ **Surface**

$$L_{xy} = m \int \sqrt{\dot{x}_\mu^2}\, d\tau \quad \longleftrightarrow \quad L_{xy} = m \int \sqrt{(\Delta X_\mu)^2}\sqrt{h}\, d\sigma\, d\tau$$

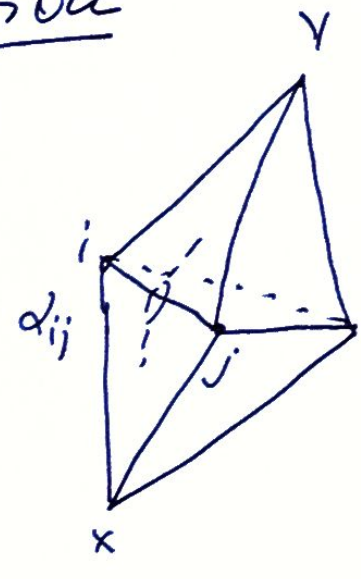
$$x_\mu = x_\mu(\tau) \quad \longleftrightarrow \quad X_\mu = X_\mu(\sigma, \tau)$$

$\longleftrightarrow$



Discretization

$\longleftrightarrow$



$$L_{xy} = m \sum_{\langle ij \rangle} l_{ij} \quad \longleftrightarrow \quad L_{xy} = m \sum_{\langle ij \rangle} l_{ij}\, |\pi - \alpha_{ij}|$$

Curvature Representation

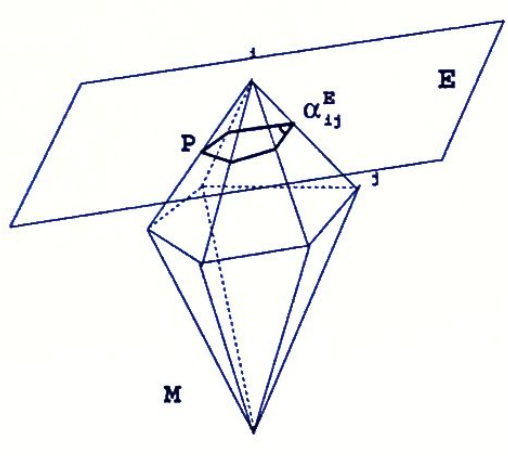
$$L = m \int \sqrt{(\Delta X_\mu)^2}\, \sqrt{h}\, d\sigma\, d\tau$$

$$= m \sum_{\langle ij \rangle} l_{ij}\, |\pi - \alpha_{ij}|$$

$$= m \int k(E)\, dE$$

{over all intersecting planes E}

$$\kappa(E) = \sum_{\langle ij \rangle} |\pi - \alpha^E_{ij}|$$



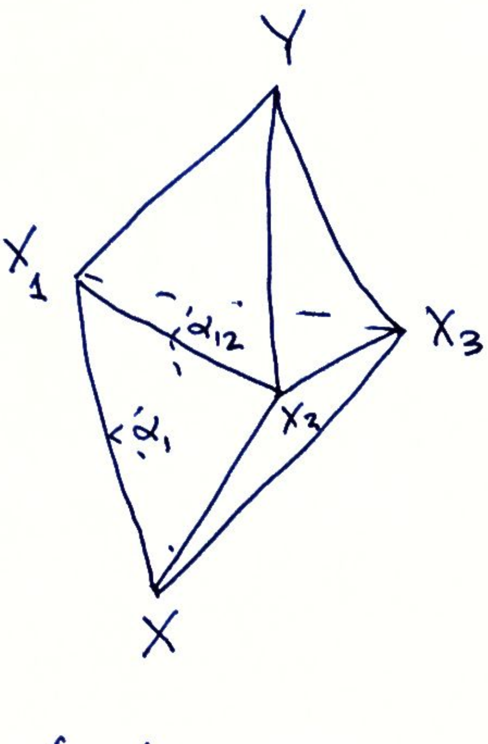
$$\kappa(E) = \sum_{\langle ij \rangle} |\pi - \alpha^E_{ij}|$$

Partition function

$$Z(\beta) = \sum_{\substack{\text{over all} \\ \text{surfaces}}} e^{-\beta \cdot \sum_{\langle ij \rangle} l_{ij} |\pi - \alpha_{ij}|}$$

$$= \sum_{\substack{\text{triangulations} \\ T \in \{T\}}} \int e^{-\beta \sum_{\langle ij \rangle} |X_i - X_j| \cdot |\pi - \alpha_{ij}|} \prod_{i \in T} dX_i$$

Ex.

$T_s =$ 

$$Z_s = \int \exp\{-\beta(|X - X_1| |\pi - \alpha_1| + |X - X_2| \cdot |\pi - \alpha_2| +$$
$$+ |X - X_3| |\pi - \alpha_3| + |X_1 - X_2| \cdot |\pi - \alpha_{12}| + \dots)\} \cdot dX\, dX_1\, dX_2\, dX_3\, dY$$

Convergence of the $Z_T(\beta)$

$$\int e^{-\beta L(T)} \prod_{i\in T} d^{\mathcal{D}} X_i = Z_T(\beta)$$

Teor: $L_T \geqslant \Delta$ diameter of the surface

Let us define

$$R = \sqrt{X_1^2 + \cdots + X_{|T|}^2} \leq \sqrt{T\Delta^2} \leq \sqrt{T}\,\Delta$$

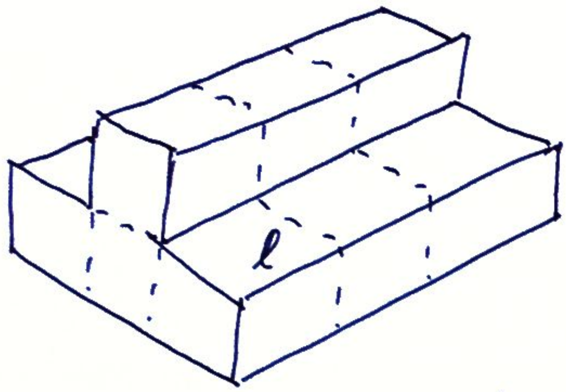
then $L_T \geqslant \Delta \geqslant \frac{R}{\sqrt{|T|}}$;

$$Z_T(\beta) = \int e^{-\beta L_T} \prod_{i\in T} d^{\mathcal{D}} X_i \leq$$

$$\leq \frac{2\pi^{\frac{\mathcal{D}|T|}{2}}}{\Gamma\left(\frac{\mathcal{D}|T|}{2}\right)} \cdot \int_0^\infty e^{-\beta\cdot\frac{R}{\sqrt{|T|}}} \cdot R^{\mathcal{D}|T|-1}\, dR$$

$$= \frac{2\pi^{\frac{\mathcal{D}|T|}{2}}}{\Gamma\left(\frac{\mathcal{D}|T|}{2}\right)} \cdot \frac{|T|^{\frac{\mathcal{D}|T|}{2}}}{\beta^{\mathcal{D}|T|}} \cdot \frac{(\mathcal{D}|T|)!}{\mathcal{D}|T|}$$

Surfaces on a lattice



$$\alpha_{ij} = \frac{\pi}{2}$$

$$|\pi - \alpha_{ij}| = \frac{\pi}{2}$$

$$H \equiv L = \sum_{\text{edges}} \ell \cdot \frac{\pi}{2} = \sum_{\text{edges}} H_{\text{edge}}$$

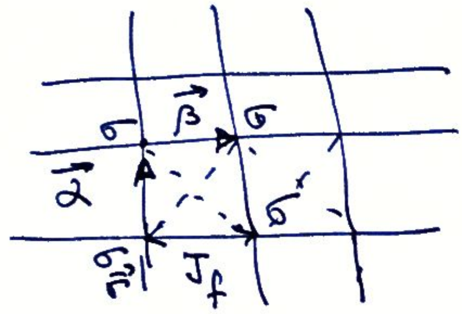
Equivalent spin System:

3D

$$H_{3D} = -2J \sum_{\vec{r},\vec{\alpha}} \sigma_{\vec{r}}\, \sigma_{\vec{r}+\vec{\alpha}} + \frac{1}{2} J \sum_{\vec{r},\vec{\alpha},\vec{\beta}} \left(\sigma_{\vec{r}}\, \sigma_{\vec{r}+\vec{\alpha}+\vec{\beta}} + \sigma_{\vec{r}+\vec{\alpha}}\, \sigma_{\vec{r}+\vec{\beta}} \right)$$

$\sigma_{\vec{r}} = \{+1, -1\}$, Vacuum degeneracy 3×2^N.

$J_{\text{ferromagnetic}} = 4\, J_{\text{antiferromagnetic}}$.

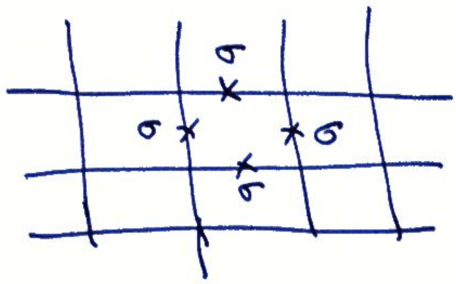


—— $J_{\text{ferr.}}$

- - - - - $J_{\text{antiferr.}}$

4D

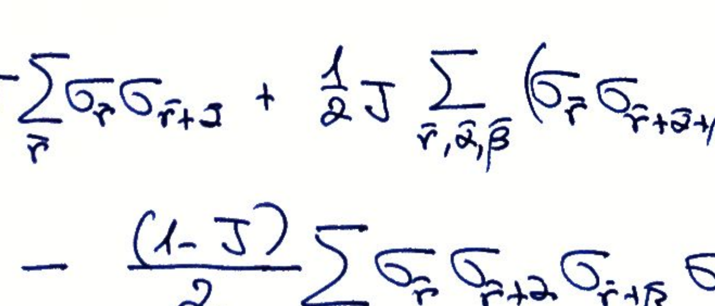
$$H_{4D} = -\frac{4}{g^2}\sum_{\text{plaquettes}}(\sigma\sigma\sigma\sigma) + \frac{1}{4g^2}\sum_{\substack{\text{right-}\\ \text{angle}\\ \text{plaquets}}}(\sigma\sigma\sigma\sigma_\alpha)^{rt}(\sigma_\alpha\sigma\sigma\sigma)$$



Partition function is:

$$Z(\beta) = \sum_{\{\sigma_{\vec r}\}} e^{-\beta H(\sigma)}$$

Intersection coupling constant.



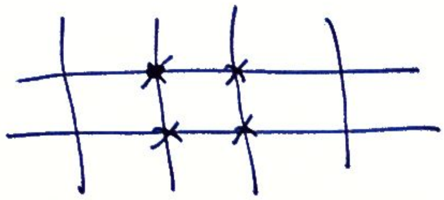
$$H_{3D} = -2J\sum_{\vec r}\sigma_{\vec r}\sigma_{\vec r+\vec\alpha} + \frac{1}{2}J\sum_{\vec r,\vec\alpha,\vec\beta}\left(\sigma_{\vec r}\sigma_{\vec r+\vec\alpha+\vec\beta} + \sigma_{\vec r+\vec\alpha}\sigma_{\vec r+\vec\beta}\right)$$

$$-\frac{(1-J)}{2}\sum_{\vec r,\vec\alpha,\vec\beta}\sigma_{\vec r}\sigma_{\vec r+\vec\alpha}\sigma_{\vec r+\vec\beta}\sigma_{\vec r+\vec\alpha+\vec\beta}.$$

Duality

$J = 0$

$$H_{3D} = -\sum \sigma_{\vec r}\,\sigma_{\vec r+\vec\alpha}\,\sigma_{\vec r+\vec\alpha+\vec\beta}\,\sigma_{\vec r+\vec\beta}$$



The system has higher symmetry

\# vacuum degeneracy $= 2^{3N}$

$$H_{3D}^{dual} = -\sum_{\langle ij\rangle}\left(u_i u_j + d_i d_j + s_i s_j\right),$$

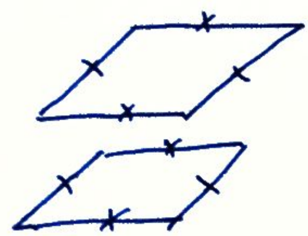
$$u_i d_i s_i = 1 .$$

3D- Ising Model.

$$H_{3D}^{Ising} = -\sum_i \sigma_i \sigma_{i+1} \iff H_{3D}^{dual} = -\sum_{\text{plaquettes}} \sigma\sigma\sigma\sigma .$$

Dual systems in 4D

$$H_{4D} = - \sum_{\substack{\text{pair of} \\ \text{parallel} \\ \text{plaquettes}}} (\sigma\sigma\sigma\sigma)''(\sigma\sigma\sigma\sigma)$$



$$H^{dual}_{3D} = \sum_{i \neq j \neq k} \Lambda_j(\xi)\,\Lambda_k(\xi)\,\Lambda_j(\xi+e_i)\,\Lambda_k(\xi+e_i)$$

$$H^{dual}_{4D} = - \sum_{\xi} \sum_{\nu \neq \mu} \Lambda_{\nu,\mu}(\xi)\,\Gamma(\xi, \xi+e_\mu)\,\Lambda_{\mu,\nu}(\xi+e_\mu)$$

$\Lambda_{\mu,\nu} \overset{\pm 1}{=} \in$ vertex ξ $\quad \mu, \nu = 1, 2, 3, 4$

$\Gamma = \pm 1 \in$ link $(\xi, \xi+e_\mu)$

Spin-glass system.

Trasfer Matrix and Spectrum in 3D

$$L = \int_{\{E\}} K(E)\, dE = \sum_{\{E_x E_y E_z\}} K(E)$$

$$Z(\beta) = \sum_{M_2} e^{-\beta \sum_{\{E\}} K(E)} =$$

$$= \sum_{M_2} \prod_{\{E_z\}} e^{-\beta k(E_z)} \prod_{\{E_x\}} e^{-\beta k(E_x)} \prod_{\{E_y\}} e^{-\beta k(E_y)} =$$

$$= \sum_{M_2} \prod_{\{E_z\}} e^{-\beta\left(k(E_z) + \ell(E_z \Delta E_{z+1})\right)}$$

$$= \sum_{\{Q_1 \cdots Q_N\}} K_\beta(Q_1, Q_2) \cdots K_\beta(Q_N, Q_1) = \mathrm{Tr}\, K_\beta^N$$

$$K_\beta(Q_1, Q_2) = \exp\left\{-\beta\left(k(Q_1) + 2\ell(Q_1 \Delta Q_2) + k(Q_2)\right)\right\}$$

Q_i - are polygon-loops

The eigenvalues of the $K_\beta(Q_1,Q_2)$

$$\sum_{\{Q_2\}} K_\beta(Q_1,Q_2)\,\psi_{Q_2} = \Lambda(\beta)\,\psi(Q_1)$$

$$Z(\beta) = \Lambda_0^N + \cdots + \Lambda_\gamma^N \quad ; \quad \gamma = 2^{N^2}$$

In cases when transfer matrix depends on symmetric difference $f(Q_1 \Delta Q_2)$

$$Q_1 \Delta Q_2 \equiv Q_1 \cup Q_2 \setminus Q_1 \cap Q_2$$

$$\sum_{\{Q_2\}} K_\beta(Q_1 \Delta Q_2)\,\psi_{Q_2} = \Lambda(\beta)\,\psi_{Q_1}$$

$$Q = Q_1 \Delta Q_2$$

$$\sum_{\{Q\}} K_\beta(Q)\,\psi(Q \Delta Q_1) = \Lambda(\beta)\,\psi(Q_1) \quad (*)$$

Introduce

$$\psi_P(Q) = e^{i\pi S(P \cap Q)}$$

eigenfunctions

$$\psi_{P_1} \cdot \psi_{P_2} = \sum_{\{Q\}} \psi_{P_1}(Q)\psi_{P_2}(Q) =$$

$$= \sum_{\{Q\}} e^{i\pi s(P_1 \cap Q) + i\pi s(P_2 \cap Q)} =$$

$$= \sum_{\{Q\}} e^{i\pi s((P_1 \Delta P_2)\cap Q)} = 2^{N^2} \delta_{P_1 P_2}.$$

$$\sum_{\{Q\}} K_\beta(Q)\psi(Q)\psi(Q_1) = \Lambda(\beta)\,\psi(Q_1)$$

$$\lambda(\beta) = \sum_{\{Q\}} K_\beta(Q)\psi(Q)$$

Eigenvalues of the transfer matrix

$$\Lambda_P(\beta) = \sum_{\{Q\}} e^{-i\pi s(P\cap Q) - \beta \ell(Q)}$$

The largest eigenvalue is 2d-Ising

$$\Lambda_\emptyset(\beta) = \sum_{\{Q\}} e^{-\beta \ell(Q)} \doteq Z(\beta)_{2D\text{-Ising}}$$

Solution of 3D Spin system in terms of coorelation functions of the 2D Ising Model:

$$\frac{\Lambda_P}{\Lambda_\phi} = \langle e^{-i\pi S(P \cap Q)} \rangle = \langle \sigma_{\vec{r}_1} \cdots \sigma_{\vec{r}_n} \rangle_{2d\text{-Ising}}$$

3D - Gonihedric $\Longleftrightarrow$ 3D - Ising.

- $K_\beta(Q_1, Q_2) = \exp\{-\beta(k(Q_1) + 2\ell(Q_1 \Delta Q_2) + k(Q_2)\}$

- $K^{Isin}_\beta(\beta) = \exp\{-\beta(\ell(Q_1) + 2S(Q_1 \Delta Q_2) + \ell(Q_2))\}$

$k(Q)$ - curvature of a polynom
$\ell(Q)$ - length of a polynom
$s(Q)$ - area of a polynom

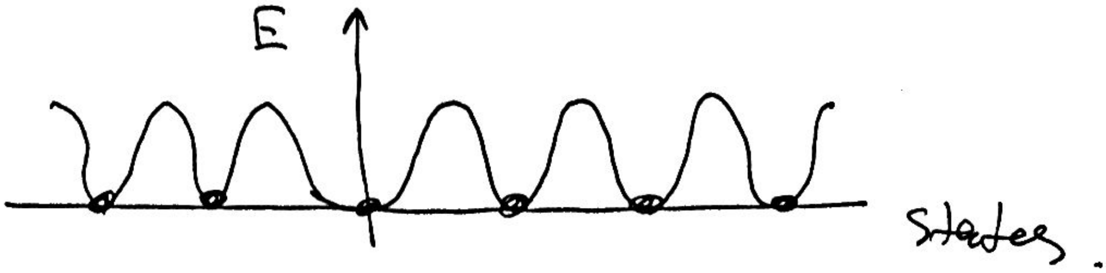
Memory Devices

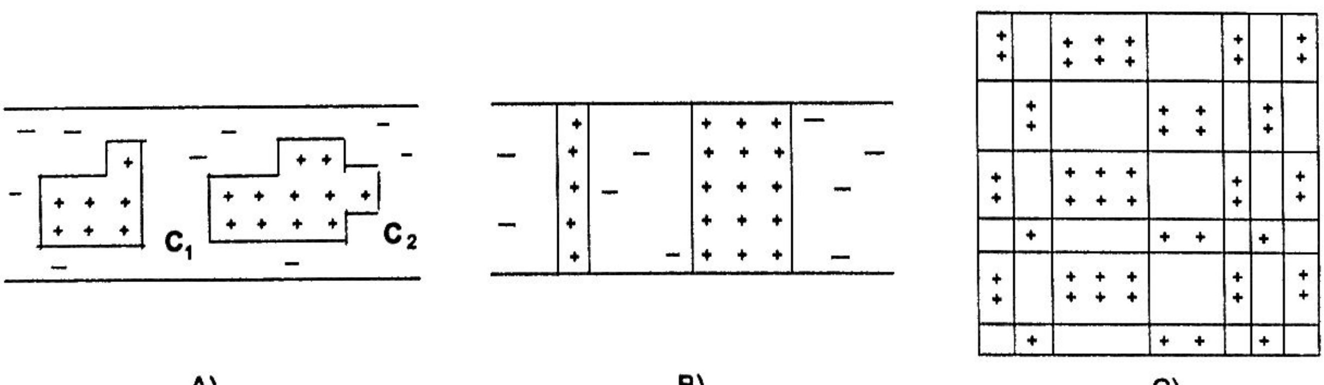
$$H^{3D}_{\text{Gonih.}} = -2kJ \sum_{\bar{r}\,\bar{a}} \sigma_{\bar{r}}\sigma_{\bar{r}+\bar{a}} \; +$$

$$+ \frac{k}{2} J \sum_{\bar{r},\bar{a},\bar{\beta}} \sigma_{\bar{r}}\sigma_{\bar{r}+\bar{a}+\bar{\beta}}$$

$$- \frac{1-k}{2} J \sum_{\bar{r},\bar{a},\bar{\beta}} \sigma_{\bar{r}}\,\sigma_{\bar{r}+\bar{a}}\,\sigma_{\bar{r}+\bar{a}+\bar{\beta}}\,\sigma_{\bar{r}+\bar{\beta}}$$

at $k=0$, the energy levels are exponentially degenerate 2^{3N}.





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