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Non-Abelian tensor gauge fields and higher-spin extension of standard model

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We suggest an extension of the gauge principle which includes non-Abelian tensor gauge fields. The invariant Lagrangian is quadratic in the field strength tensors and describes interaction of charged tensor gauge bosons of arbitrary large integer spin $1, 2, \ldots$. Non-Abelian tensor gauge fields can be viewed as a unique gauge field with values in the infinite-dimensional current algebra associated with compact Lie group. The full Lagrangian exhibits also enhanced local gauge invariance with double number of gauge parameters which allows to eliminate all negative norm states of the nonsymmetric second-rank tensor gauge field, which describes therefore two polarizations of helicity-two massless charged tensor gauge boson and the helicity-zero "axion" The geometrical interpretation of the enhanced gauge symmetry with double number of gauge parameters is not yet known.

Contents

1 Introduction

The non-Abelian local gauge invariance, which was formulated by Yang and Mills in [1], requires that all interactions must be invariant under independent rotations of internal charges at all space-time points. The gauge principle allows very little arbitrariness: the interaction of matter fields, which carry non-commuting internal charges, and the nonlinear self-interaction of gauge bosons are essentially fixed by the requirement of local gauge invariance, very similar to the self-interaction of gravitons in general relativity.

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It is therefore appealing to extend the gauge principle, which was elevated by Yang and Mills to a powerful constructive principle, so that it will define the interaction of matter fields which carry not only non-commutative internal charges, but also arbitrary large spins. It seems that this will naturally lead to a theory in which fundamental forces will be mediated by integer-spin gauge quanta and that the Yang-Mills vector gauge boson will become a member of a bigger family of tensor gauge bosons.

In the recent papers [2–6] we extended the gauge principle so that it enlarges the Yang-Mills group of local gauge transformations and defines interaction of tensor gauge bosons of arbitrary large integer spins. The extended non-Abelian gauge transformations of the tensor gauge fields form *a new large group which has a natural geometrical interpretation in terms of extended current algebra associated with compact Lie group G*. On this large group one can define field strength tensors, which are transforming homogeneously with respect to the extended gauge transformations. The invariant Lagrangian is quadratic in the field strength tensors and describes interaction of tensor gauge fields of arbitrary large integer spin $1, 2, \ldots$. It was also demonstrated that the total Lagrangian exhibits enhanced local gauge invariance with double number of gauge parameters. This allows to eliminate all negative norm states of the nonsymmetric second rank tensor gauge field $A_{\mu\lambda}$, which describes therefore two polarizations of helicity-two massless charged tensor gauge boson and the helicity-zero "axion".

The early investigation of higher-spin representations of the Poincaré algebra and of the corresponding field equations is due to Majorana, Dirac and Wigner. The theory of massive particles of higher spin was further developed by Fierz and Pauli [7] and Rarita and Schwinger [8]. The Lagrangian and S-matrix formulations of *free field theory* of massive and massless fields with higher spin have been completely constructed in [9–16]. The problem of *introducing interaction* appears to be much more complex [17] and met enormous difficulties for spin fields higher than two [18]. The first positive result in this direction was the light-front construction of the cubic interaction term for the massless field of helicity $\pm\lambda$ in [19,20].

2 Extended gauge transformations

In our approach the gauge fields are defined as rank-$(s+1)$ tensors [2–4]

$$A^a_{\mu\lambda_1\ldots\lambda_s}(x), \qquad s = 0, 1, 2, \ldots$$

and are totally symmetric with respect to the indices $\lambda_1 \ldots \lambda_s$. *A priory* the tensor fields have no symmetries with respect to the first index μ. The index a numerates the generators L^a of the Lie algebra $\breve{g}$ of a *compact*[1] Lie group G. One can think of these tensor fields as appear in the expansion of the extended gauge field $\mathcal{A}_\mu(x, e)$ over the tangent space-like unit vector e_λ [2–4]

$$\mathcal{A}_\mu(x, e) = \sum_{s=0}^{\infty} A^a_{\mu\lambda_1\ldots\lambda_s}(x)\, L^a e_{\lambda_1} \ldots e_{\lambda_s}. \tag{1}$$

The gauge field $A^a_{\mu\lambda_1\ldots\lambda_s}$ carry indices $a, \lambda_1, \ldots, \lambda_s$ labeling the generators of *extended current algebra $\mathcal{G}$ associated with compact Lie group G.* It has infinite many generators $L^a_{\lambda_1\ldots\lambda_s} = L^a e_{\lambda_1} \ldots e_{\lambda_s}$ and the corresponding algebra is given by the commutator

$$\left[L^a_{\lambda_1\ldots\lambda_s}, L^b_{\rho_1\ldots\rho_k}\right] = i f^{abc} L^c_{\lambda_1\ldots\lambda_s\rho_1\ldots\rho_k}. \tag{2}$$

The extended non-Abelian gauge transformations of the tensor gauge fields are defined by the following equations [3]:

$$\delta A^a_\mu = \left(\delta^{ab}\partial_\mu + g f^{acb} A^c_\mu\right) \xi^b\,, \tag{3}$$

[1] The algebra $\breve{g}$ possesses an orthogonal basis in which the structure constant f^{abc} are totally antisymmetric.

$$\delta A^a_{\mu\nu} = \left(\delta^{ab}\partial_\mu + g f^{acb} A^c_\mu\right) \xi^b_\nu + g f^{acb} A^c_{\mu\nu} \xi^b,$$
$$\delta A^a_{\mu\nu\lambda} = \left(\delta^{ab}\partial_\mu + g f^{acb} A^c_\mu\right) \xi^b_{\nu\lambda} + g f^{acb} \left(A^c_{\mu\nu}\xi^b_\lambda + A^c_{\mu\lambda}\xi^b_\nu + A^c_{\mu\nu\lambda}\xi^b\right),$$
$$\ldots \cdot \ldots$$

These extended gauge transformations generate a closed algebraic structure. To see that, one should compute the commutator of two extended gauge transformations δ_η and δ_ξ of parameters η and ξ. The commutator of two transformations can be expressed in the form [3]

$$[\,\delta_\eta, \delta_\xi]\; A_{\mu\lambda_1\lambda_2\ldots\lambda_s} \;=\; -ig\, \delta_\zeta A_{\mu\lambda_1\lambda_2\ldots\lambda_s} \tag{4}$$

and is again an extended gauge transformation with the gauge parameters $\{\zeta\}$ which are given by the matrix commutators

$$\zeta = [\eta, \xi] \tag{5}$$
$$\zeta_{\lambda_1} = [\eta, \xi_{\lambda_1}] + [\eta_{\lambda_1}, \xi]$$
$$\zeta_{\nu\lambda} = [\eta, \xi_{\nu\lambda}] + [\eta_\nu, \xi_\lambda] + [\eta_\lambda, \xi_\nu] + [\eta_{\nu\lambda}, \xi],$$
$$\ldots \cdot \ldots$$

The generalized field strengths are defined as [3]

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc}\, A^b_\mu\, A^c_\nu, \tag{6}$$
$$G^a_{\mu\nu,\lambda} = \partial_\mu A^a_{\nu\lambda} - \partial_\nu A^a_{\mu\lambda} + g f^{abc} \left(A^b_\mu\, A^c_{\nu\lambda} + A^b_{\mu\lambda}\, A^c_\nu\right),$$
$$G^a_{\mu\nu,\lambda\rho} = \partial_\mu A^a_{\nu\lambda\rho} - \partial_\nu A^a_{\mu\lambda\rho} + g f^{abc} \left(A^b_\mu\, A^c_{\nu\lambda\rho} + A^b_{\mu\lambda}\, A^c_{\nu\rho} + A^b_{\mu\rho}\, A^c_{\nu\lambda} + A^b_{\mu\lambda\rho}\, A^c_\nu\right),$$
$$\ldots \cdot \ldots$$

and transform homogeneously with respect to the extended gauge transformations (3). The field strength tensors are antisymmetric in their first two indices and are totaly symmetric with respect to the rest of the indices.

The inhomogeneous extended gauge transformation (3) induces the homogeneous gauge transformation of the corresponding field strength (6) of the form [3]

$$\delta G^a_{\mu\nu} = g f^{abc} G^b_{\mu\nu} \xi^c \tag{7}$$
$$\delta G^a_{\mu\nu,\lambda} = g f^{abc} \left(G^b_{\mu\nu,\lambda}\xi^c + G^b_{\mu\nu}\xi^c_\lambda\right),$$
$$\delta G^a_{\mu\nu,\lambda\rho} = g f^{abc} \left(G^b_{\mu\nu,\lambda\rho}\xi^c + G^b_{\mu\nu,\lambda}\xi^c_\rho + G^b_{\mu\nu,\rho}\xi^c_\lambda + G^b_{\mu\nu}\xi^c_{\lambda\rho}\right)$$
$$\ldots \cdot \ldots,$$

The field strength tensors are antisymmetric in their first two indices and are totaly symmetric with respect to the rest of the indices. The symmetry properties of the field strength $G^a_{\mu\nu,\lambda_1\ldots\lambda_s}$ remain invariant in the course of this transformation. By induction the entire construction can be generalized to include tensor fields of any rank s [2,3].

3 First gauge invariant lagrangian

The gauge invariant Lagrangian now can be formulated in the form [3]

$$\mathcal{L}_{s+1} = -\tfrac{1}{4}\, G^a_{\mu\nu,\lambda_1\ldots\lambda_s}\, G^a_{\mu\nu,\lambda_1\ldots\lambda_s} + \ldots$$

$$= -\frac{1}{4} \sum_{i=0}^{2s} a_i^s \, G^a_{\mu\nu,\lambda_1\ldots\lambda_i} \, G^a_{\mu\nu,\lambda_{i+1}\ldots\lambda_{2s}} \left(\sum_{p's} \eta^{\lambda_{i_1}\lambda_{i_2}} \ldots \eta^{\lambda_{i_{2s-1}}\lambda_{i_{2s}}} \right) , \tag{8}$$

where the sum $\sum_p$ runs over all nonequal permutations of $i's$, in total $(2s-1)!!$ terms. For the low values of $s = 0, 1, 2, \ldots$ the numerical coefficients

$$a_i^s = \frac{s!}{i!(2s-i)!}$$

are: $a_0^0 = 1$; $a_1^1 = 1, a_0^1 = a_2^1 = 1/2$; $a_2^2 = 1/2, a_1^2 = a_3^2 = 1/3, a_0^2 = a_4^2 = 1/12$; and so on. In order to describe fixed rank-$(s+1)$ gauge field one should have at disposal all gauge fields up to the rank $2s+1$. In order to make all tensor gauge fields dynamical one should add the corresponding kinetic terms. Thus the invariant Lagrangian describing dynamical tensor gauge bosons of all ranks has the form

$$\mathcal{L} = \sum_{s=1}^{\infty} g_s \mathcal{L}_s \; . \tag{9}$$

The first three terms of the invariant Lagrangian have the following form [3]:

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \ldots = -\tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - \tfrac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda} - \tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda}$$
$$-\tfrac{1}{4} G^a_{\mu\nu,\lambda\rho} G^a_{\mu\nu,\lambda\rho} - \tfrac{1}{8} G^a_{\mu\nu,\lambda\lambda} G^a_{\mu\nu,\rho\rho} - \tfrac{1}{2} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda\rho\rho} - \tfrac{1}{8} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda\rho\rho} + \ldots , \tag{10}$$

where the first term is the Yang-Mills Lagrangian and the second and the third ones describe the tensor gauge fields $A^a_{\mu\nu}, A^a_{\mu\nu\lambda}$ and so on. It is important that: i) *the Lagrangian does not contain higher derivatives of tensor gauge fields ii) all interactions take place through the three- and four-particle exchanges with dimensionless coupling constant iii) the complete Lagrangian contains all higher-rank tensor gauge fields and should not be truncated.*

4 Geometrical interpretation

Let us consider a possible geometrical interpretation of the above construction. Introducing tangent space-time unit vector e_μ and considering it as a second variable we can introduce the extended gauge parameter $\xi^a_{\lambda_1\ldots\lambda_s}(x)$ and the generators $L^a_{\lambda_1\ldots\lambda_s} = L^a e_{\lambda_1} \ldots e_{\lambda_s}$ [2–4]

$$\xi(x,e) = \sum_{s=0}^{\infty} \xi^a_{\lambda_1\ldots\lambda_s}(x) \; L^a e_{\lambda_1} \ldots e_{\lambda_s} \tag{11}$$

and define the gauge transformation of the extended gauge field $\mathcal{A}_\mu(x,e)$ as in (3)

$$\mathcal{A}'_\mu(x,e) = U(\xi)\mathcal{A}_\mu(x,e)U^{-1}(\xi) - \frac{i}{g}\partial_\mu U(\xi)\, U^{-1}(\xi), \tag{12}$$

where the unitary transformation matrix is given by the expression $U(\xi) = exp\{ig\xi(x,e)\}$. This allows to construct the extended field strength tensor of the form (6)

$$\mathcal{G}_{\mu\nu}(x,e) = \partial_\mu \mathcal{A}_\nu(x,e) - \partial_\nu \mathcal{A}_\mu(x,e) - ig\left[\mathcal{A}_\mu(x,e)\; \mathcal{A}_\nu(x,e)\right] \tag{13}$$

using the commutator of the covariant derivatives $\nabla^{ab}_\mu = (\partial_\mu - ig\mathcal{A}_\mu(x,e))^{ab}$ of a standard form $[\nabla_\mu, \nabla_\nu]^{ab} = g f^{acb} \mathcal{G}^c_{\mu\nu}$, so that

$$\mathcal{G}'_{\mu\nu}(x,e)) = U(\xi)\mathcal{G}_{\mu\nu}(x,e)U^{-1}(\xi). \tag{14}$$

The invariant Lagrangian density is given by the expression

$$\mathcal{L}(x,e) = \mathcal{G}^a_{\mu\nu}(x,e)\mathcal{G}^a_{\mu\nu}(x,e), \tag{15}$$

as one can be convinced computing its variation with respect to the extended gauge transformation (3),(12) and (7),(14)

$$\delta\mathcal{L}(x,e) = 2\mathcal{G}^a_{\mu\nu}(x,e)\, gf^{acb}\, \mathcal{G}^c_{\mu\nu}(x,e)\, \xi^b(x,e) = 0.$$

The Lagrangian density (15) allows to extract *gauge invariant, totally symmetric, tensor densities* $\mathcal{L}_{\lambda_1\ldots\lambda_s}(x)$ using expansion with respect to the vector variable e

$$\mathcal{L}(x,e) = \sum_{s=0}^{\infty} \mathcal{L}_{\lambda_1\ldots\lambda_s}(x)\, e_{\lambda_1}\ldots e_{\lambda_s}. \tag{16}$$

In particular the expansion term which is quadratic in powers of e is (see the next section for explicit variation (22))

$$(\mathcal{L}_2)_{\lambda\rho} = -\tfrac{1}{4}G^a_{\mu\nu,\lambda}G^a_{\mu\nu,\rho} - \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu,\lambda\rho} \tag{17}$$

and defines a unique Lorentz invariant Lagrangian which can be constructed from the above tensor, that is the Lagrangian $\mathcal{L}_2$

$$\mathcal{L}_2 = -\tfrac{1}{4}G^a_{\mu\nu,\lambda}G^a_{\mu\nu,\lambda} - \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu,\lambda\lambda}.$$

The whole construction can be viewed as an extended vector bundle X on which the gauge field $\mathcal{A}^a_\mu(x,e)$ is a connection. The gauge field $A^a_{\mu\lambda_1\ldots\lambda_s}$ carry indices $a, \lambda_1, \ldots, \lambda_s$ which label the generators of the *extended current algebra $\mathcal{G}$ associated with the compact Lie group G*. It has infinite many generators $L^a_{\lambda_1\ldots\lambda_s} = L^a e_{\lambda_1}\ldots e_{\lambda_s}$ with commutator

$$\left[L^a_{\lambda_1\ldots\lambda_s}, L^b_{\rho_1\ldots\rho_k}\right] = if^{abc}L^c_{\lambda_1\ldots\lambda_s\rho_1\ldots\rho_k}. \tag{18}$$

Thus we have vector bundle whose structure group is an extended gauge group $\mathcal{G}$ with group elements $U(\xi) = exp(i\xi(e))$, where $\xi(e) = \sum_s \xi^a_{\lambda_1\ldots\lambda_s} L^a e_{\lambda_1}\ldots e_{\lambda_s}$ and the composition law (5). In contrast, in Kac-Moody current algebra the generators depend on the complex variable $L^a_n = L^a z^n$ (see also [21])

$$\left[L^a_n, L^b_m\right] = if^{abc}L^c_{n+m}.$$

In the next section we shall see that, there exist a second invariant Lagrangian $\mathcal{L}^{'}$ which can be constructed in terms of extended field strength tensors (6) and the total Lagrangian is a linear sum of the two Lagrangians $c\,\mathcal{L} + c^{'}\,\mathcal{L}^{'}$.

5 Second gauge invariant lagrangian

Indeed the Lagrangian (8), (9) and (10) is not the most general Lagrangian which can be constructed in terms of the above field strength tensors (6) and (13). As we shall see there exists a second invariant Lagrangian $\mathcal{L}^{'}$ (19), (20) and (21) which can be constructed in terms of extended field strength tensors (6) and the total Lagrangian is a linear sum of the two Lagrangians $c\ \mathcal{L} + c^{'}\ \mathcal{L}^{'}$. In particular for the second-rank tensor gauge field $A^a_{\mu\lambda}$ the total Lagrangian is a sum of two Lagrangians $c\mathcal{L}_2 + c^{'}\mathcal{L}^{'}_2$ and, with specially chosen coefficients $\{c, c^{'}\}$, it exhibits an enhanced gauge invariance (27),(35) with double number of gauge parameters, which allows to eliminate negative norm polarizations of the nonsymmetric second-rank tensor gauge field $A_{\mu\lambda}$. The geometrical interpretation of the enhanced gauge symmetry with double number of gauge parameters is not yet known.

Let us consider the gauge invariant tensor density of the form

$$\mathcal{L}^{'}_{\rho_1\rho_2}(x,e) = \frac{1}{4}\mathcal{G}^a_{\mu\rho_1}(x,e)\mathcal{G}^a_{\mu\rho_2}(x,e). \tag{19}$$

It is gauge invariant because its variation is also equal to zero

$$\delta\mathcal{L}^{'}_{\rho_1\rho_2}(x,e) = \tfrac{1}{4}gf^{acb}\,\mathcal{G}^c_{\mu\rho_1}(x,e)\,\xi^b(x,e)\mathcal{G}^a_{\mu\rho_2}(x,e) + \tfrac{1}{4}\mathcal{G}^a_{\mu\rho_1}(x,e)\,gf^{acb}\,\mathcal{G}^c_{\mu\rho_2}(x,e)\,\xi^b(x,e) = 0.$$

The Lagrangian density (19) generate the second series of *gauge invariant tensor densities* $(\mathcal{L}^{'}_{\rho_1\rho_2})_{\lambda_1\ldots\lambda_s}(x)$ when we expand it in powers of the vector variable e

$$\mathcal{L}^{'}_{\rho_1\rho_2}(x,e) = \sum_{s=0}^{\infty}(\mathcal{L}^{'}_{\rho_1\rho_2})_{\lambda_1\ldots\lambda_s}(x)\,e_{\lambda_1}\ldots e_{\lambda_s}. \tag{20}$$

Using contraction of these tensor densities the gauge invariant Lagrangians can be formulated in the form

$$\begin{aligned}\mathcal{L}^{'}_{s+1} &= \tfrac{1}{4}\,G^a_{\mu\lambda_1,\lambda_2\ldots\lambda_{s+1}}\,G^a_{\mu\lambda_2,\lambda_1\ldots\lambda_{s+1}} + \ldots\\ &= \tfrac{1}{8}\sum_{i=1}^{2s+1} a^s_{i-1}\,G^a_{\mu\lambda_1,\lambda_2\ldots\lambda_i}\,G^a_{\mu\lambda_{i+1},\lambda_{i+2}\ldots\lambda_{2s+2}}\left(\sum_{p's}^{'}\eta^{\lambda_{i_1}\lambda_{i_2}}\ldots\eta^{\lambda_{i_{2s+1}}\lambda_{i_{2s+2}}}\right),\end{aligned} \tag{21}$$

where the sum $\sum_p^{'}$ runs over all nonequal permutations of $i's$, with exclusion of the terms which contain $\eta^{\lambda_1,\lambda_{i+1}}$.

It is also instructive to construct these Lagrangian densities explicitly. The invariance of the first Lagrangian $\mathcal{L}_2$

$$\mathcal{L}_2 = -\tfrac{1}{4}G^a_{\mu\nu,\lambda}G^a_{\mu\nu,\lambda} - \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu,\lambda\lambda}$$

in (8), (9) and (10) was demonstrated in [3] by calculating its variation with respect to the gauge transformation (3) and (7). Indeed, its explicit variation is equal to zero

$$\begin{aligned}\delta\mathcal{L}_2 = &-\tfrac{1}{4}G^a_{\mu\nu,\lambda}gf^{abc}\left(G^b_{\mu\nu,\lambda}\xi^c + G^b_{\mu\nu}\xi^c_\lambda\right) - \tfrac{1}{4}gf^{abc}\left(G^a_{\mu\nu,\lambda}\xi^c + G^b_{\mu\nu}\xi^c_\lambda\right)G^b_{\mu\nu,\lambda}\\ &-\tfrac{1}{4}gf^{abc}G^b_{\mu\nu}\xi^c G^a_{\mu\nu,\lambda\lambda}\\ &-\tfrac{1}{4}G^a_{\mu\nu}gf^{abc}\left(G^b_{\mu\nu,\lambda\lambda}\xi^c + G^b_{\mu\nu,\lambda}\xi^c_\lambda + G^b_{\mu\nu,\lambda}\xi^c_\lambda + G^b_{\mu\nu}\xi^c_{\lambda\lambda}\right) = 0.\end{aligned} \tag{22}$$

As we have seen above a general consideration shows that the Lagrangian $\mathcal{L}_2$ is not a unique one and that there exist a second invariant Lagrangian $\mathcal{L}^{'}_2$. Let us construct this Lagrangian density explicitly. First notice that there exist additional Lorentz invariant quadratic forms which can be constructed by the corresponding field strength tensors. They are [2]

$$G^a_{\mu\nu,\lambda}G^a_{\mu\lambda,\nu}, \qquad G^a_{\mu\nu,\nu}G^a_{\mu\lambda,\lambda}, \qquad G^a_{\mu\nu}G^a_{\mu\lambda,\nu\lambda}.$$

Calculating the variation of each of these terms with respect to the gauge transformation (3) and (7) one can get convinced that a particular linear combination

$$\mathcal{L}^{'}_2 = \tfrac{1}{4}G^a_{\mu\nu,\lambda}G^a_{\mu\lambda,\nu} + \tfrac{1}{4}G^a_{\mu\nu,\nu}G^a_{\mu\lambda,\lambda} + \tfrac{1}{2}G^a_{\mu\nu}G^a_{\mu\lambda,\nu\lambda} \tag{23}$$

forms an invariant Lagrangian and coincide with (21) for s=1. Indeed the variation of the Lagrangian $\mathcal{L}_2^{'}$ under the gauge transformation (7) is equal to zero:

$$\begin{aligned}\delta\mathcal{L}_2^{'} = &+\tfrac{1}{4}G^a_{\mu\nu,\lambda} g f^{abc}\left(G^b_{\mu\lambda,\nu}\xi^c + G^b_{\mu\lambda}\xi^c_\nu\right) + \tfrac{1}{4} g f^{abc}\left(G^b_{\mu\nu,\lambda}\xi^c + G^b_{\mu\nu}\xi^c_\lambda\right) G^a_{\mu\lambda,\nu}\\ &+\tfrac{1}{2}G^a_{\mu\nu,\nu} g f^{abc}\left(G^b_{\mu\lambda,\lambda}\xi^c + G^b_{\mu\lambda}\xi^c_\lambda\right)\\ &+\tfrac{1}{2} g f^{abc} G^b_{\mu\nu}\xi^c G^a_{\mu\lambda,\nu\lambda}\\ &+\tfrac{1}{2}G^a_{\mu\nu} g f^{abc}\left(G^b_{\mu\lambda,\nu\lambda}\xi^c + G^b_{\mu\lambda,\nu}\xi^c_\lambda + G^b_{\mu\lambda,\lambda}\xi^c_\nu + G^b_{\mu\lambda}\xi^c_{\nu\lambda}\right) = 0.\end{aligned}$$

As a result we have two invariant Lagrangians $\mathcal{L}_2$ and $\mathcal{L}_2^{'}$ and the general Lagrangian is a linear combination of these two Lagrangians $\mathcal{L}_2 + c\mathcal{L}_2^{'}$, where c is an arbitrary constant.

Our aim now is to demonstrate that if $c = 1$ *then we shall have enhanced local gauge invariance* (27),(35) *of the Lagrangian* $\mathcal{L}_2 + \mathcal{L}_2^{'}$ *with double number of gauge parameters. This allows to eliminate all negative norm states of the nonsymmetric second-rank tensor gauge field* $A^a_{\mu\lambda}$*, which describes therefore two polarizations of helicity-two massless charged tensor gauge boson and of the helicity-zero "axion".*

6 Enhancement of extended gauge transformations

Indeed, let us consider the situation at the linearized level when the gauge coupling constant g is equal to zero. The free part of the $\mathcal{L}_2$ Lagrangian is

$$\mathcal{L}_2^{\text{free}} = \tfrac{1}{2}A^a_{\alpha\acute{\alpha}}\left(\eta_{\alpha\gamma}\eta_{\acute{\alpha}\acute{\gamma}}\partial^2 - \eta_{\acute{\alpha}\acute{\gamma}}\partial_\alpha\partial_\gamma\right)A^a_{\gamma\acute{\gamma}} = \tfrac{1}{2}A^a_{\alpha\acute{\alpha}}H_{\alpha\acute{\alpha}\gamma\acute{\gamma}}A^a_{\gamma\acute{\gamma}},$$

where the quadratic form in the momentum representation has the form

$$H_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = -(k^2\eta_{\alpha\gamma} - k_\alpha k_\gamma)\eta_{\acute{\alpha}\acute{\gamma}} = -H_{\alpha\gamma}(k)\eta_{\acute{\alpha}\acute{\gamma}},$$

is obviously invariant with respect to the gauge transformation $\delta A^a_{\mu\lambda} = \partial_\mu\xi^a_\lambda$, but it is not invariant with respect to the alternative gauge transformations $\delta A^a_{\mu\lambda} = \partial_\lambda\eta^a_\mu$. This can be seen, for example, from the following relations in the momentum representation:

$$k_\alpha H_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = 0, \qquad k_{\acute{\alpha}} H_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = -\left(k^2\eta_{\alpha\gamma} - k_\alpha k_\gamma\right)k_{\acute{\gamma}} \neq 0. \tag{24}$$

Let us consider now the free part of the second Lagrangian

$$\begin{aligned}\mathcal{L}_2^{'\text{free}} = &\tfrac{1}{4}A^a_{\alpha\acute{\alpha}}\big(-\eta_{\alpha\acute{\gamma}}\eta_{\acute{\alpha}\gamma}\partial^2 - \eta_{\alpha\acute{\alpha}}\eta_{\gamma\acute{\gamma}}\partial^2 + \eta_{\alpha\acute{\gamma}}\partial_{\acute{\alpha}}\partial_\gamma + \eta_{\acute{\alpha}\gamma}\partial_\alpha\partial_{\acute{\gamma}} + \eta_{\alpha\acute{\alpha}}\partial_\gamma\partial_{\acute{\gamma}} +\\ &+\eta_{\gamma\acute{\gamma}}\partial_\alpha\partial_{\acute{\alpha}} - 2\eta_{\alpha\gamma}\partial_{\acute{\alpha}}\partial_{\acute{\gamma}}\big)A^a_{\gamma\acute{\gamma}} = \tfrac{1}{2}A^a_{\alpha\acute{\alpha}}H^{'}_{\alpha\acute{\alpha}\gamma\acute{\gamma}}A^a_{\gamma\acute{\gamma}},\end{aligned} \tag{25}$$

where

$$H^{'}_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = \tfrac{1}{2}(\eta_{\alpha\acute{\gamma}}\eta_{\acute{\alpha}\gamma} + \eta_{\alpha\acute{\alpha}}\eta_{\gamma\acute{\gamma}})k^2 - \tfrac{1}{2}(\eta_{\alpha\acute{\gamma}}k_{\acute{\alpha}}k_\gamma + \eta_{\acute{\alpha}\gamma}k_\alpha k_{\acute{\gamma}} + \eta_{\alpha\acute{\alpha}}k_\gamma k_{\acute{\gamma}} + \eta_{\gamma\acute{\gamma}}k_\alpha k_{\acute{\alpha}} - 2\eta_{\alpha\gamma}k_{\acute{\alpha}}k_{\acute{\gamma}}).$$

It is again invariant with respect to the gauge transformation $\delta A^a_{\mu\lambda} = \partial_\mu\xi^a_\lambda$, but it is not invariant with respect to the gauge transformations $\delta A^a_{\mu\lambda} = \partial_\lambda\eta^a_\mu$ as one can see from analogous relations

$$k_\alpha H^{'}_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = 0, \qquad k_{\acute{\alpha}} H^{'}_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = \left(k^2\eta_{\alpha\gamma} - k_\alpha k_\gamma\right)k_{\acute{\gamma}} \neq 0. \tag{26}$$

As it is obvious from (24) and (26), the total Lagrangian $\mathcal{L}_2^{\text{free}} + \mathcal{L}_2^{'\text{free}}$ now poses new enhanced invariance with respect to the larger, eight parameter, gauge transformations

$$\delta A^a_{\mu\lambda} = \partial_\mu\xi^a_\lambda + \partial_\lambda\eta^a_\mu + \ldots, \tag{27}$$

where ξ^a_λ and η^a_μ are eight arbitrary functions, because

$$k_\alpha \left(H_{\alpha\acute{\alpha}\gamma\acute{\gamma}} + H^{'}_{\alpha\acute{\alpha}\gamma\acute{\gamma}} \right) = 0, \qquad k_{\acute{\alpha}} \left(H_{\alpha\acute{\alpha}\gamma\acute{\gamma}} + H^{'}_{\alpha\acute{\alpha}\gamma\acute{\gamma}} \right) = 0. \tag{28}$$

Thus our free part of the Lagrangian is

$$\begin{aligned}\mathcal{L}_2^{\text{tot free}} = & -\tfrac{1}{2}\partial_\mu A^a_{\nu\lambda}\partial_\mu A^a_{\nu\lambda} + \tfrac{1}{2}\partial_\mu A^a_{\nu\lambda}\partial_\nu A^a_{\mu\lambda} + \\ & + \tfrac{1}{4}\partial_\mu A^a_{\nu\lambda}\partial_\mu A^a_{\lambda\nu} - \tfrac{1}{4}\partial_\mu A^a_{\nu\lambda}\partial_\lambda A^a_{\mu\nu} - \tfrac{1}{4}\partial_\nu A^a_{\mu\lambda}\partial_\mu A^a_{\lambda\nu} + \tfrac{1}{4}\partial_\nu A^a_{\mu\lambda}\partial_\lambda A^a_{\mu\nu} \\ & + \tfrac{1}{4}\partial_\mu A^a_{\nu\nu}\partial_\mu A^a_{\lambda\lambda} - \tfrac{1}{2}\partial_\mu A^a_{\nu\nu}\partial_\lambda A^a_{\mu\lambda} + \tfrac{1}{4}\partial_\nu A^a_{\mu\nu}\partial_\lambda A^a_{\mu\lambda}\end{aligned} \tag{29}$$

or, in equivalent form, it is

$$\begin{aligned}\mathcal{L}_2^{\text{tot free}} = \tfrac{1}{2} A^a_{\alpha\acute{\alpha}} \{ & \left(\eta_{\alpha\gamma}\eta_{\acute{\alpha}\acute{\gamma}} - \tfrac{1}{2}\eta_{\alpha\acute{\gamma}}\eta_{\acute{\alpha}\gamma} - \tfrac{1}{2}\eta_{\alpha\acute{\alpha}}\eta_{\gamma\acute{\gamma}}\right)\partial^2 - \eta_{\acute{\alpha}\acute{\gamma}}\partial_\alpha\partial_\gamma - \eta_{\alpha\gamma}\partial_{\acute{\alpha}}\partial_{\acute{\gamma}} + \\ & + \tfrac{1}{2}\left(\eta_{\alpha\acute{\gamma}}\partial_{\acute{\alpha}}\partial_\gamma + \eta_{\acute{\alpha}\gamma}\partial_\alpha\partial_{\acute{\gamma}} + \eta_{\alpha\acute{\alpha}}\partial_\gamma\partial_{\acute{\gamma}} + \eta_{\gamma\acute{\gamma}}\partial_\alpha\partial_{\acute{\alpha}}\right)\} A^a_{\gamma\acute{\gamma}}\end{aligned} \tag{30}$$

and is invariant with respect to the larger gauge transformations $\delta A^a_{\mu\lambda} = \partial_\mu \xi^a_\lambda + \partial_\lambda \eta^a_\mu$, where ξ^a_λ and η^a_μ are eight arbitrary functions. In the momentum representation the quadratic form is

$$\begin{aligned}H^{\text{tot}}_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = & \left(-\eta_{\alpha\gamma}\eta_{\acute{\alpha}\acute{\gamma}} + \tfrac{1}{2}\eta_{\alpha\acute{\gamma}}\eta_{\acute{\alpha}\gamma} + \tfrac{1}{2}\eta_{\alpha\acute{\alpha}}\eta_{\gamma\acute{\gamma}}\right)k^2 + \eta_{\alpha\gamma}k_{\acute{\alpha}}k_{\acute{\gamma}} + \eta_{\acute{\alpha}\acute{\gamma}}k_\alpha k_\gamma \\ & - \tfrac{1}{2}\left(\eta_{\alpha\acute{\gamma}}k_{\acute{\alpha}}k_\gamma + \eta_{\acute{\alpha}\gamma}k_\alpha k_{\acute{\gamma}} + \eta_{\alpha\acute{\alpha}}k_\gamma k_{\acute{\gamma}} + \eta_{\gamma\acute{\gamma}}k_\alpha k_{\acute{\alpha}}\right).\end{aligned} \tag{31}$$

Let us consider also the symmetries of the remaining two terms in the full Lagrangian $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}^{'}_2$. They have the form

$$-\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu,\lambda\lambda} + \tfrac{1}{2}G^a_{\mu\nu}G^a_{\mu\lambda,\nu\lambda}.$$

The part which is quadratic in fields has the form

$$\begin{aligned}\mathcal{L}_2^{\text{free}} = \tfrac{1}{2}A^a_\alpha\{ \quad & +\eta_{\gamma'\gamma''} - \eta_{\alpha\gamma}\partial_{\gamma'}\partial_{\gamma''} \\ & -\tfrac{1}{2}\left(\eta_{\gamma\gamma''}\partial^2 - \partial_\gamma\partial_{\gamma''}\right)\eta_{\alpha\gamma'} - \tfrac{1}{2}\left(\eta_{\gamma\gamma'}\partial^2 - \partial_\gamma\partial_{\gamma'}\right)\eta_{\alpha\gamma''} \\ & +\tfrac{1}{2}\eta_{\gamma\gamma''}\partial_\alpha\partial_{\gamma'} + \tfrac{1}{2}\eta_{\gamma\gamma'}\partial_\alpha\partial_{\gamma''} \;\}A^a_{\gamma\gamma'\gamma''} \\ = \tfrac{1}{2}A^a_\alpha H_{\alpha\gamma\gamma'\gamma''}A^a_{\gamma\gamma'\gamma''},\end{aligned} \tag{32}$$

where the quadratic form in the momentum representation is

$$\begin{aligned}H_{\alpha\gamma\gamma'\gamma''}(k) = & -\left(\eta_{\alpha\gamma}k^2 - k_\alpha k_\gamma\right)\eta_{\gamma'\gamma''} + \eta_{\alpha\gamma}k_{\gamma'}k_{\gamma''} \\ & +\tfrac{1}{2}\left(\eta_{\gamma\gamma''}k^2 - k_\gamma k_{\gamma''}\right)\eta_{\alpha\gamma'} - \tfrac{1}{2}\eta_{\gamma\gamma''}k_\alpha k_{\gamma'} \\ & +\tfrac{1}{2}\left(\eta_{\gamma\gamma'}k^2 - k_\gamma k_{\gamma'}\right)\eta_{\alpha\gamma''} - \tfrac{1}{2}\eta_{\gamma\gamma'}k_\alpha k_{\gamma''}.\end{aligned} \tag{33}$$

As one can see all divergences are equal to zero

$$k_\alpha H_{\alpha\gamma\gamma'\gamma''}(k) = k_\gamma H_{\alpha\gamma\gamma'\gamma''}(k) = k_{\gamma'} H_{\alpha\gamma\gamma'\gamma''}(k) = k_{\gamma''} H_{\alpha\gamma\gamma'\gamma''}(k) = 0. \tag{34}$$

This result means that the quadratic part of the full Lagrangian $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}^{'}_2$ is invariant under the following local gauge transformations

$$\tilde{\delta}_\eta A^a_\mu = \partial_\mu \eta^a + \ldots$$

$$\tilde{\delta}_\eta A^a_{\mu\nu} = \partial_\nu \eta^a_\mu + \ldots, \qquad (35)$$
$$\tilde{\delta}_\eta A^a_{\mu\nu\lambda} = \partial_\nu \eta^a_{\mu\lambda} + \partial_\lambda \eta^a_{\mu\nu} + \ldots$$
$$\ldots \cdot \ldots,$$

in addition to the initial local gauge transformations (3)

$$\delta_\xi A^a_\mu = \partial_\mu \xi^a + \ldots$$
$$\delta_\xi A^a_{\mu\nu} = \partial_\mu \xi^a_\nu + \ldots$$
$$\delta_\xi A^a_{\mu\nu\lambda} = \partial_\mu \xi^a_{\nu\lambda} + \ldots \qquad (36)$$

It is important to known how the transformation (35) looks like when the gauge coupling constant is not equal to zero. The existence of the full transformation is guaranteed by the conservation of the corresponding currents (43), (44) and (45). At the moment we can only guess the full form of the second local gauge transformation requiring the closure of the corresponding algebra. The extension we have found has the form [2]:

$$\tilde{\delta}_\eta A^a_\mu = \left(\delta^{ab}\partial_\mu + g f^{acb} A^c_\mu\right) \eta^b, \qquad (37)$$
$$\tilde{\delta}_\eta A^a_{\mu\nu} = \left(\delta^{ab}\partial_\nu + g f^{acb} A^c_\nu\right) \eta^b_\mu + g f^{acb} A^c_{\mu\nu}\eta^b,$$
$$\tilde{\delta}_\eta A^a_{\mu\nu\lambda} = \left(\delta^{ab}\partial_\nu + g f^{acb} A^c_\nu\right) \eta^b_{\mu\lambda} + \left(\delta^{ab}\partial_\lambda + g f^{acb} A^c_\lambda\right) \eta^b_{\mu\nu} + g f^{acb}\left(A^c_{\mu\nu}\eta^b_\lambda + A^c_{\mu\lambda}\eta^b_\nu + A^c_{\mu\nu\lambda}\eta^b\right)$$
$$\ldots\ldots\ldots,$$

and forms a closed algebraic structure. The composition law of the gauge parameters $\{\eta, \eta_\nu, \eta_{\nu\lambda}, \ldots\}$ is the same as in (5).

7 Total lagrangian and equation of motion

In summary, we have the following Lagrangian for the lower-rank tensor gauge fields:

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_2^{'} = -\tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} \qquad (38)$$
$$-\tfrac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\nu,\lambda} - \tfrac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu,\lambda\lambda}$$
$$+\tfrac{1}{4} G^a_{\mu\nu,\lambda} G^a_{\mu\lambda,\nu} + \tfrac{1}{4} G^a_{\mu\nu,\nu} G^a_{\mu\lambda,\lambda} + \tfrac{1}{2} G^a_{\mu\nu} G^a_{\mu\lambda,\nu\lambda}.$$

Let us consider the equations of motion which follow from this Lagrangian for the vector gauge field A^a_ν:

$$\nabla^{ab}_\mu G^b_{\mu\nu} + \tfrac{1}{2}\nabla^{ab}_\mu \left(G^b_{\mu\nu,\lambda\lambda} + G^b_{\nu\lambda,\mu\lambda} + G^b_{\lambda\mu,\nu\lambda}\right) + g f^{acb} A^c_{\mu\lambda} G^b_{\mu\nu,\lambda} \qquad (39)$$
$$-\tfrac{1}{2} g f^{acb}\left(A^c_{\mu\lambda} G^b_{\mu\lambda,\nu} + A^c_{\mu\lambda} G^b_{\lambda\nu,\mu} + A^c_{\lambda\lambda} G^b_{\mu\nu,\mu} + A^c_{\mu\nu} G^b_{\mu\lambda,\lambda}\right)$$
$$+\tfrac{1}{2} g f^{acb}\left(A^c_{\mu\lambda\lambda} G^b_{\mu\nu} + A^c_{\mu\mu\lambda} G^b_{\nu\lambda} + A^c_{\mu\nu\lambda} G^b_{\lambda\mu}\right) = 0$$

and for the second-rank tensor gauge field $A^a_{\nu\lambda}$:

$$\nabla^{ab}_\mu G^b_{\mu\nu,\lambda} - \tfrac{1}{2}\left(\nabla^{ab}_\mu G^b_{\mu\lambda,\nu} + \nabla^{ab}_\mu G^b_{\lambda\nu,\mu} + \nabla^{ab}_\lambda G^b_{\mu\nu,\mu} + \eta_{\nu\lambda}\nabla^{ab}_\mu G^b_{\mu\rho,\rho}\right)$$
$$+ g f^{acb} A^c_{\mu\lambda} G^b_{\mu\nu} - \tfrac{1}{2} g f^{acb}\left(A^c_{\mu\nu} G^b_{\mu\lambda} + A^c_{\mu\mu} G^b_{\lambda\nu} + A^c_{\lambda\mu} G^b_{\mu\nu} + \eta_{\nu\lambda} A^c_{\mu\rho} G^b_{\mu\rho}\right) = 0. \qquad (40)$$

The variation of the action with respect to the third-rank gauge field $A^a_{\nu\lambda\rho}$ will give the equations

$$\eta_{\lambda\rho}\nabla^{ab}_{\mu}G^b_{\mu\nu} - \frac{1}{2}\left(\eta_{\nu\rho}\nabla^{ab}_{\mu}G^b_{\mu\lambda} + \eta_{\lambda\nu}\nabla^{ab}_{\mu}G^b_{\mu\rho}\right) + \tfrac{1}{2}\left(\nabla^{ab}_{\rho}G^b_{\nu\lambda} + \nabla^{ab}_{\lambda}G^b_{\nu\rho}\right) = 0. \tag{41}$$

Representing these system of equations in the form

$$\partial_\mu F^a_{\mu\nu} + \tfrac{1}{2}\partial_\mu\left(F^a_{\mu\nu,\lambda\lambda} + F^a_{\nu\lambda,\mu\lambda} + F^a_{\lambda\mu,\nu\lambda}\right) = j^a_\nu \tag{42}$$

$$\partial_\mu F^a_{\mu\nu,\lambda} - \tfrac{1}{2}\left(\partial_\mu F^a_{\mu\lambda,\nu} + \partial_\mu F^a_{\lambda\nu,\mu} + \partial_\lambda F^a_{\mu\nu,\mu} + \eta_{\nu\lambda}\partial_\mu F^a_{\mu\rho,\rho}\right) = j^a_{\nu\lambda}$$

$$\eta_{\lambda\rho}\partial_\mu F^a_{\mu\nu} - \frac{1}{2}\left(\eta_{\nu\rho}\partial_\mu F^a_{\mu\lambda} + \eta_{\nu\lambda}\partial_\mu F^a_{\mu\rho}\right) + \tfrac{1}{2}\left(\partial_\rho F^a_{\nu\lambda} + \partial_\lambda F^a_{\nu\rho}\right) = j^a_{\nu\lambda\rho},$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu,\ F^a_{\mu\nu,\lambda} = \partial_\mu A^a_{\nu\lambda} - \partial_\nu A^a_{\mu\lambda},\ F^a_{\mu\nu,\lambda\rho} = \partial_\mu A^a_{\nu\lambda\rho} - \partial_\nu A^a_{\mu\lambda\rho}$, we can find the corresponding conserved currents

$$\begin{aligned} j^a_\nu = &-gf^{abc}A^b_\mu G^c_{\mu\nu} - gf^{abc}\partial_\mu\left(A^b_\mu A^c_\nu\right) \\ &-\tfrac{1}{2}gf^{abc}A^b_\mu\left(G^c_{\mu\nu,\lambda\lambda} + G^c_{\nu\lambda,\mu\lambda} + G^c_{\lambda\mu,\nu\lambda}\right) - \tfrac{1}{2}\partial_\mu\left(I^a_{\mu\nu,\lambda\lambda} + I^a_{\nu\lambda,\mu\lambda} + I^a_{\lambda\mu,\nu\lambda}\right) \\ &-gf^{abc}A^b_{\mu\lambda}G^c_{\mu\nu,\lambda} + \tfrac{1}{2}gf^{abc}\left(A^b_{\mu\lambda}G^c_{\mu\lambda,\nu} + A^b_{\mu\lambda}G^c_{\lambda\nu,\mu} + A^b_{\lambda\lambda}G^c_{\mu\nu,\mu} + A^b_{\mu\nu}G^c_{\mu\lambda,\lambda}\right) \\ &-\tfrac{1}{2}gf^{abc}\left(A^b_{\mu\lambda\lambda}G^c_{\mu\nu} + A^b_{\lambda\mu\lambda}G^c_{\nu\mu} + A^b_{\mu\lambda\nu}G^c_{\lambda\mu}\right), \end{aligned} \tag{43}$$

where $I^a_{\mu\nu,\lambda\rho} = gf^{abc}\left(A^b_\mu\ A^c_{\nu\lambda\rho} + A^b_{\mu\lambda}\ A^c_{\nu\rho} + A^b_{\mu\rho}\ A^c_{\nu\lambda} + A^b_{\mu\lambda\rho}\ A^c_\nu\right)$ and

$$\begin{aligned} j^a_{\nu\lambda} = &-gf^{abc}A^b_\mu G^c_{\mu\nu,\lambda} + \tfrac{1}{2}gf^{abc}\left(A^b_\mu G^c_{\mu\lambda,\nu} + A^b_\mu G^c_{\lambda\nu,\mu} + A^b_\lambda G^c_{\mu\nu,\mu} + \eta_{\nu\lambda}A^b_\mu G^c_{\mu\rho,\rho}\right) \\ &- gf^{abc}A^b_{\mu\lambda}G^c_{\mu\nu} + \tfrac{1}{2}gf^{abc}\left(A^b_{\mu\nu}G^c_{\mu\lambda} + A^b_{\lambda\mu}G^c_{\mu\nu} + A^b_{\mu\mu}G^c_{\lambda\nu} + \eta_{\nu\lambda}A^b_{\mu\rho}G^c_{\mu\rho}\right) \\ &- gf^{abc}\partial_\mu\left(A^b_\mu A^c_{\nu\lambda} + A^b_{\mu\lambda}A^c_\nu\right) + \tfrac{1}{2}gf^{abc}[\partial_\mu\left(A^b_\mu A^c_{\lambda\nu} + A^b_{\mu\nu}A^c_\lambda\right) + \partial_\mu\left(A^b_\lambda A^c_{\nu\mu} + A^b_{\lambda\mu}A^c_\nu\right) \\ &+ \partial_\lambda\left(A^b_\mu A^b_{\nu\mu} + A^b_{\mu\mu}A^c_\nu\right) + \eta_{\nu\lambda}\partial_\mu\left(A^b_\mu A^b_{\rho\rho} + A^b_{\mu\rho}A^c_\rho\right)], \end{aligned} \tag{44}$$

$$\begin{aligned} j^a_{\nu\lambda\rho} = &-\eta_{\lambda\rho}\ gf^{abc}A^b_\mu G^c_{\mu\nu} + \tfrac{1}{2}gf^{abc}\left(\eta_{\nu\rho}A^b_\mu G^c_{\mu\lambda} + \eta_{\nu\lambda}A^b_\mu G^c_{\mu\rho} - A^b_\rho G^c_{\nu\lambda} - A^b_\lambda G^c_{\nu\rho}\right) \\ &- \eta_{\lambda\rho}\ gf^{abc}\partial_\mu\left(A^b_\mu A^c_\nu\right) + \tfrac{1}{2}gf^{abc}\left[\partial_\mu\left(\eta_{\nu\lambda}A^b_\mu A^c_\rho + \eta_{\nu\rho}A^b_\mu A^c_\lambda\right) - \partial_\lambda\left(A^b_\nu A^c_\rho\right) - \partial_\rho\left(A^b_\nu A^c_\lambda\right)\right]. \end{aligned} \tag{45}$$

Thus

$$\begin{aligned} &\partial_\nu j^a_\nu = 0, \\ &\partial_\nu j^a_{\nu\lambda} = 0, \qquad \partial_\lambda j^a_{\nu\lambda} = 0, \\ &\partial_\nu j^a_{\nu\lambda\rho} = 0, \qquad \partial_\lambda j^a_{\nu\lambda\rho} = 0, \qquad \partial_\rho j^a_{\nu\lambda\rho} = 0, \end{aligned} \tag{46}$$

because, as we demonstrated above, the partial derivatives of the l.h.s. of the eqs. (42) are equal to zero (see eqs. (28) and eqs. (34)).

8 Linearized equations and propagating modes

At the linearized level, when the gauge coupling constant g is equal to zero, the equations of motion (40) for the second-rank tensor gauge fields will take the form

$$\partial^2 \left(A^a_{\nu\lambda} - \tfrac{1}{2}A^a_{\lambda\nu}\right) - \partial_\nu\partial_\mu \left(A^a_{\mu\lambda} - \tfrac{1}{2}A^a_{\lambda\mu}\right) - \partial_\lambda\partial_\mu \left(A^a_{\nu\mu} - \tfrac{1}{2}A^a_{\mu\nu}\right) + \partial_\nu\partial_\lambda \left(A^a_{\mu\mu} - \tfrac{1}{2}A^a_{\mu\mu}\right) + \tfrac{1}{2}\eta_{\nu\lambda}\left(\partial_\mu\partial_\rho A^a_{\mu\rho} - \partial^2 A^a_{\mu\mu}\right) = 0 \tag{47}$$

and, as we shall see below, they describe the propagation of massless particles of spin 2 and spin 0. It is also easy to see that for the symmetric part of the tensor gauge field $\left(A^a_{\nu\lambda} + A^a_{\lambda\nu}\right)/2$ our equation reduces to the well known Fierz-Pauli-Schwinger-Chang-Singh-Hagen-Fronsdal equation

$$\partial^2 A_{\nu\lambda} - \partial_\nu\partial_\mu A_{\mu\lambda} - \partial_\lambda\partial_\mu A_{\mu\nu} + \partial_\nu\partial_\lambda A_{\mu\mu} + \eta_{\nu\lambda}\left(\partial_\mu\partial_\rho A_{\mu\rho} - \partial^2 A_{\mu\mu}\right) = 0, \tag{48}$$

which describes the propagation of massless tensor boson with two physical polarizations, the $\lambda = \pm 2$ helicity states. For the antisymmetric part $(A^a_{\nu\lambda} - A^a_{\lambda\nu})/2$ the equation reduces to the form

$$\partial^2 A_{\nu\lambda} - \partial_\nu\partial_\mu A_{\mu\lambda} + \partial_\lambda\partial_\mu A_{\mu\nu} = 0 \tag{49}$$

and describes the propagation of massless scalar boson with one physical polarization, the $\lambda = 0$ helicity state.

We can find out now how many propagating degrees of freedom describe the system of eqs. (47) in the classical theory. Taking the trace of the equation (47) we shall get

$$\partial_\mu\partial_\rho A^a_{\mu\rho} - \partial^2 A^a_{\rho\rho} = 0, \tag{50}$$

and the equation (47) takes the form

$$\partial^2 \left(A^a_{\nu\lambda} - \tfrac{1}{2}A^a_{\lambda\nu}\right) - \partial_\nu\partial_\mu \left(A^a_{\mu\lambda} - \tfrac{1}{2}A^a_{\lambda\mu}\right) - \partial_\lambda\partial_\mu \left(A^a_{\nu\mu} - \tfrac{1}{2}A^a_{\mu\nu}\right) + \tfrac{1}{2}\partial_\nu\partial_\lambda A^a_{\mu\mu} = 0. \tag{51}$$

Using the gauge invariance (27) we can impose the Lorentz invariant supplementary conditions on the second-rank gauge fields $A_{\mu\lambda}$: $\partial_\mu A^a_{\mu\lambda} = a_\lambda, \quad \partial_\lambda A^a_{\mu\lambda} = b_\mu$, where a_λ and b_μ are arbitrary functions, or one can suggest alternative supplementary conditions in which the quadratic form (29), (30), (31) is diagonal:

$$\partial_\mu A^a_{\mu\lambda} - \tfrac{1}{2}\partial_\lambda A^a_{\mu\mu} = 0, \qquad \partial_\lambda A^a_{\mu\lambda} - \tfrac{1}{2}\partial_\mu A^a_{\lambda\lambda} = 0. \tag{52}$$

In this gauge the equation (51) has the form

$$\partial^2 A^a_{\nu\lambda} = 0 \tag{53}$$

and in the momentum representation $A_{\mu\nu}(x) = e_{\mu\nu}(k)e^{ikx}$ from eq. (53) it follows that $k^2 = 0$ and we have *massless particles*.

For the symmetric part of the tensor field $A^a_{\mu\lambda}$ the supplementary conditions (52) are equivalent to the harmonic gauge

$$\partial_\mu \left(A^a_{\mu\lambda} + A^a_{\lambda\mu}\right) - \tfrac{1}{2}\partial_\lambda \left(A^a_{\mu\mu} + A^a_{\mu\mu}\right) = 0, \tag{54}$$

and the residual gauge transformations are defined by the gauge parameters $\xi^a_\lambda + \eta^a_\lambda$ which should satisfy the equation

$$\partial^2 \left(\xi^a_\lambda + \eta^a_\lambda\right) = 0. \tag{55}$$

Thus imposing the harmonic gauge (54) and using the residual gauge transformations (55) one can see that the number of propagating physical polarizations which are described by the symmetric part of the tensor field $A^a_{\mu\lambda}$ are given by two helicity states $\lambda = \pm 2$ multiplied by the dimension of the group G (a=1,...,N).

For the anisymmetric part of the tensor field $A^a_{\mu\lambda}$ the supplementary conditions (52) are equivalent to the Lorentz gauge

$$\partial_\mu (A^a_{\mu\lambda} - A^a_{\lambda\mu}) = 0 \tag{56}$$

and together with the equation of motion they describe the propagation of one physical polarization of helicity $\lambda = 0$ multiplied by the dimension of the group G (a=1,...,N).

Thus we have seen that the extended gauge symmetry (27) with eight gauge parameters is sufficient to exclude all negative norm polarizations from the spectrum of the second-rank *nonsymmetric tensor gauge field* $A_{\mu\lambda}$ which describes now the propagation of three physical modes of helicities ± 2 and 0.

In the Lorentz-like gauge and with the traceless condition we shall get

$$H^{fix}_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k) = \left(\eta_{\alpha\gamma}\eta_{\acute{\alpha}\acute{\gamma}} - \tfrac{1}{2}\eta_{\alpha\acute{\alpha}}\eta_{\gamma\acute{\gamma}}\right)(-k^2)$$

and the propagator $\Delta_{\gamma\acute{\gamma}\lambda\acute{\lambda}}(k)$ from the equation $H^{fix}_{\alpha\acute{\alpha}\gamma\acute{\gamma}}(k)\Delta_{\gamma\acute{\gamma}\lambda\acute{\lambda}}(k) = \eta_{\alpha\lambda}\eta_{\acute{\alpha}\acute{\lambda}}$, thus

$$\Delta_{\gamma\acute{\gamma}\lambda\acute{\lambda}}(k) = -\frac{\eta_{\gamma\lambda}\eta_{\acute{\gamma}\acute{\lambda}} - \frac{1}{2}\eta_{\gamma\acute{\gamma}}\eta_{\lambda\acute{\lambda}}}{k^2 - i\varepsilon}\,. \tag{57}$$

The corresponding residue can be represented as a sum

$$\eta_{\gamma\lambda}\eta_{\acute{\gamma}\acute{\lambda}} - \tfrac{1}{2}\eta_{\gamma\acute{\gamma}}\eta_{\lambda\acute{\lambda}} = +\tfrac{1}{2}\left(\eta_{\gamma\lambda}\eta_{\acute{\gamma}\acute{\lambda}} + \eta_{\gamma\acute{\lambda}}\eta_{\acute{\gamma}\lambda} - \eta_{\gamma\acute{\gamma}}\eta_{\lambda\acute{\lambda}}\right) + \tfrac{1}{2}\left(\eta_{\gamma\lambda}\eta_{\acute{\gamma}\acute{\lambda}} - \eta_{\gamma\acute{\lambda}}\eta_{\acute{\gamma}\lambda}\right). \tag{58}$$

The first term describes the $\lambda = \pm 2$ helicity states and is represented by the symmetric part of the polarization tensor $e_{\mu\lambda}$, the second term describes $\lambda = 0$ helicity state and is represented by its antisymmetric part. Indeed, for the massless case, when k_μ is aligned along the third axis, $k_\mu = (k, 0, 0, k)$, we have two independent polarizations of the helicity-2 particle and spin-zero *axion*

$$e^1_{\mu\lambda} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0,0,\ 0,0 \\ 0,1,\ 0,0 \\ 0,0,-1,0 \\ 0,0,\ 0,0 \end{pmatrix}, \quad e^2_{\mu\lambda} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0,0,0,0 \\ 0,0,1,0 \\ 0,1,0,0 \\ 0,0,0,0 \end{pmatrix}, \quad e^A_{\mu\lambda} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0,\ 0,0,0 \\ 0,\ 0,1,0 \\ 0,-1,0,0 \\ 0,\ 0,0,0 \end{pmatrix}, \tag{59}$$

with the property that $e^1_{\gamma\acute{\gamma}}e^1_{\lambda\acute{\lambda}} + e^2_{\alpha\acute{\alpha}}e^2_{\gamma\acute{\gamma}} \simeq \frac{1}{2}(\eta_{\gamma\lambda}\eta_{\acute{\gamma}\acute{\lambda}} + \eta_{\gamma\acute{\lambda}}\eta_{\acute{\gamma}\lambda} - \eta_{\gamma\acute{\gamma}}\eta_{\lambda\acute{\lambda}})$ and $e^A_{\gamma\acute{\gamma}}e^A_{\lambda\acute{\lambda}} \simeq \frac{1}{2}(\eta_{\gamma\lambda}\eta_{\acute{\gamma}\acute{\lambda}} - \eta_{\gamma\acute{\lambda}}\eta_{\acute{\gamma}\lambda})$. The symbol $\simeq$ means that the equation holds up to longitudinal terms.

Thus the general second-rank tensor gauge field with 16 components $A_{\mu\lambda}$ describes in this theory three physical propagating massless polarizations.

9 Higher-spin extension of electroweak theory

Let us consider the possible extension of the standard model of electroweak interactions which follows from the above generalization. In the first model which we shall consider only the $SU(2)_L$ group will be extended to higher spins, but not the $U(1)_Y$ group. The $W^\pm, Z$ gauge bosons will receive their higher-spin descendence

$$(W^\pm, Z)_\mu, \qquad \left(\tilde{W}^\pm, \tilde{Z}\right)_{\mu\lambda}, \ldots$$

and the doublet of complex Higgs scalars will appear together with their higher-spin partners:

$$\begin{pmatrix}\phi^+\\ \phi^o\end{pmatrix}, \quad \begin{pmatrix}\phi^+\\ \phi^o\end{pmatrix}, \quad \begin{pmatrix}\phi^+\\ \phi^o\end{pmatrix}_{\lambda\rho}, \ldots \quad Y=+1.$$

The Lagrangian which describes the interaction of the tensor gauge bosons with scalar fields and tensor bosons is:

$$\begin{aligned}\mathcal{L} = & -\tfrac{1}{4}G^i_{\mu\nu}G^i_{\mu\nu} - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \left(\partial_\mu + \frac{ig^{'}}{2}B_\mu + \frac{ig}{2}\tau^i A^i_\mu\right)\phi^\dagger \left(\partial_\mu - \frac{ig^{'}}{2}B_\mu - \frac{ig}{2}\tau^i A^i_\mu\right)\phi \\ & -\tfrac{1}{4}G^i_{\mu\nu,\lambda}G^i_{\mu\nu,\lambda} - \tfrac{1}{4}G^i_{\mu\nu}G^i_{\mu\nu,\lambda\lambda} + \tfrac{1}{4}G^i_{\mu\nu,\lambda}G^i_{\mu\lambda,\nu} + \tfrac{1}{4}G^i_{\mu\nu,\nu}G^i_{\mu\lambda,\lambda} + \tfrac{1}{2}G^i_{\mu\nu}G^i_{\mu\lambda,\nu\lambda} \\ & + \frac{g^2}{4}\phi^\dagger\tau^i A^i_{\mu\lambda}\tau^j A^j_{\mu\lambda}\phi + \nabla_\mu\phi^\dagger_\lambda\,\nabla_\mu\phi_\lambda + \tfrac{1}{2}\nabla_\mu\phi^\dagger_{\lambda\lambda}\,\nabla_\mu\phi + \tfrac{1}{2}\nabla_\mu\phi^\dagger\,\nabla_\mu\phi_{\lambda\lambda} \\ & -ig\nabla_\mu\phi^\dagger\,A_{\mu\lambda}\phi_\lambda + ig\phi^\dagger_\lambda A_{\mu\lambda}\,\nabla_\mu\phi - ig\nabla_\mu\phi^\dagger_\lambda\,A_{\mu\lambda}\phi + ig\phi^\dagger A_{\mu\lambda}\,\nabla_\mu\phi_\lambda - \\ & -\tfrac{1}{2}ig\nabla_\mu\phi^\dagger\,A_{\mu\lambda\lambda}\phi + \tfrac{1}{2}ig\phi^\dagger A_{\mu\lambda\lambda}\,\nabla_\mu\phi\,, \end{aligned} \tag{60}$$

where $\nabla_\mu = \partial_\mu - \frac{ig^{'}}{2}YB_\mu - igT^iA^i_\mu$, Y is hypercharge, Q is charge, $Q = T_3 + Y/2$, and for isospinor fields $T^i = \tau^i/2$. The three terms in the first line represent the standard electroweak model and the rest of the terms - its higher-spin generalization. Therefore all parameters of the standard model are incorporated in the extension. The first term in the third line will generate the masses of the tensor $\tilde{W}^\pm, \tilde{Z}$ gauge bosons:

$$\tfrac{1}{8}g^2\eta^2\left[\left(A^3_{\mu\lambda}\right)^2 + 2A^+_{\mu\lambda}A^-_{\mu\lambda}\right], \tag{61}$$

when the scalar fields acquire the vacuum expectation value η: $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \eta+\chi(x)\end{pmatrix}$ and

$$\tilde{Z}_{\mu\lambda} = A^3_{\mu\lambda}, \qquad \tilde{W}^\pm_{\mu\lambda} = \frac{1}{\sqrt{2}}\left(A^1_{\mu\lambda} \pm iA^2_{\mu\lambda}\right),$$

Thus all three intermediate spin-2 bosons will acquire the mass proportional to $m_{\tilde{W},\tilde{Z}} = \frac{1}{2}g\eta = m_W$. The rest of the terms describe the interaction between "old" and new particles. One should also introduce the Yukawa self-interaction for the bosons ϕ_λ in order to make them massive.

Let us consider the fermion sector of the extended electroweak model. One should note that the interaction of tensor gauge bosons with fermions is not as usual as one could expect. Indeed, let us now analyze the interaction with new spinor-tensor leptons

$$L = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e\\ e\end{pmatrix}, \quad L_\lambda = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e\\ e\end{pmatrix}_\lambda, \quad L_{\lambda\rho} = \tfrac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e\\ e\end{pmatrix}_{\lambda\rho} \ldots \quad Y=-1.$$

All these left-handed states have hypercharge $Y = -1$ and the only right-handed state

$$R = \tfrac{1}{2}(1-\gamma_5)e, \qquad Y=-2$$

has the hypercharge $Y = -2$. The corresponding Lagrangian will take the form

$$\begin{aligned}\mathcal{L}_F = & \bar{L}\,\not\nabla L + \bar{R}\,\not\nabla R + \bar{L}_\lambda\,\not\nabla L_\lambda + \tfrac{1}{2}\bar{L}\,\not\nabla L_{\lambda\lambda} + \tfrac{1}{2}\bar{L}_{\lambda\lambda}\,\not\nabla L + g\bar{L}_\lambda\,\not A_\lambda L \\ & + g\bar{L}\,\not A_\lambda L_\lambda + \tfrac{1}{2}g\bar{L}\,\not A_{\lambda\lambda}L,\end{aligned}$$

where the first two terms describe the standard electroweak interaction of vector gauge bosons with standard spin-1/2 leptons, the next three terms describe the interaction of the vector gauge bosons with new leptons of the spin 3/2 and finally the last three terms describe the interaction of the new tensor gauge bosons $\tilde{W}^{\pm}, \tilde{Z}$ with standard spin-1/2 and spin-3/2 leptons.

The new interaction vertices generate decay of the standard vector gauge bosons through the channels

$$\gamma, Z \to e_{3/2} + \bar{e}_{3/2}, \quad \gamma, Z \to \nu_{3/2} + \bar{\nu}_{3/2}, \quad W \to \nu_{3/2} + e_{3/2}, \quad W \to \nu_{3/2} + e_{3/2} \,,$$

where a pair of new leptons is created. The observability of these channels depends on the masses of the new leptons. This information is encoded into the Yukawa couplings, as it takes place for the standard leptons of the spin 1/2. We can only say that they are large enough not to be seen at low energies, but are predicted to be visible at higher-energy experiments.

The decay reactions of the new tensor gauge bosons $\tilde{W}^{\pm}, \tilde{Z}$ can take place through the channels

$$\tilde{Z} \to e_{3/2} + \bar{e}_{1/2}, \quad \tilde{Z} \to \nu_{3/2} + \bar{\nu}_{1/2}, \quad \tilde{W} \to \nu_{1/2} + e_{3/2}, \quad \tilde{W} \to \nu_{3/2} + e_{1/2}. \tag{62}$$

The main feature of these processes is that they create a pair which consists of a standard lepton $e_{1/2}$ and of a new lepton $e_{3/2}$ of the spin 3/2. Because in all these reactions there always participates a new lepton, they may take place also at large enough energies, but it is impossible to predict the threshold energy because we do not know the corresponding Yukawa couplings. The situation with Yukawa couplings is the same as it is in the standard model. There is no decay channels of the new tensor bosons only into the standard leptons, as one can see from the Lagrangian. Therefore it is also impossible to create tensor gauge bosons directly in $e^+ + e^-$ annihilation, but they can appear in the decay of the Z

$$e^+ + e^- \to Z \to \tilde{W}^+ + \tilde{W}^- \tag{63}$$

and will afterwards decay through the channels discussed above (62) $\tilde{W} \to \nu + \tilde{e}$ or $\tilde{W} \to \tilde{\nu} + e$. *It seems that reaction* (63)*, predicted by the generalized theory, is the most appropriate candidate which could be tested in the experiment*. The details will be given in the forthcoming publication.

We did not consider the tensor extension of the $U(1)_Y$ in the first place, because in that case we shall have the massless spin-2 descendent of the photon, which most probably should be associated with the graviton. The right-handed sector should be enlarged in the following way:

$$R = \tfrac{1}{2}(1-\gamma_5)e, \quad R_\lambda = \tfrac{1}{2}(1-\gamma_5)e_\lambda, \quad R_{\lambda\rho} = \tfrac{1}{2}(1-\gamma_5)e_{\lambda\rho}, \ldots, \quad Y = -2,$$

and the Lagrangian will take the form

$$\begin{aligned}\mathcal{L}_F &= \bar{L}\, \nabla\!\!\!\!/ L + \bar{R}\, \nabla\!\!\!\!/ R \\ &+ \bar{L}_\lambda\, \nabla\!\!\!\!/ L_\lambda + \tfrac{1}{2}\bar{L}\, \nabla\!\!\!\!/ L_{\lambda\lambda} + \tfrac{1}{2}\bar{L}_{\lambda\lambda}\, \nabla\!\!\!\!/ L + g\bar{L}_\lambda\, A\!\!\!/_\lambda L + g\bar{L}\, A\!\!\!/_\lambda L_\lambda + \tfrac{1}{2}g\bar{L}\, A\!\!\!/_{\lambda\lambda} L \\ &+ \bar{R}_\lambda\, \nabla\!\!\!\!/ R_\lambda + \tfrac{1}{2}\bar{R}\, \nabla\!\!\!\!/ R_{\lambda\lambda} + \tfrac{1}{2}\bar{R}_{\lambda\lambda}\, \nabla\!\!\!\!/ R + g'\bar{R}_\lambda\, B\!\!\!/_\lambda R + g'\bar{R}\, B\!\!\!/_\lambda R_\lambda + \tfrac{1}{2}g'\bar{R}\, B\!\!\!/_{\lambda\lambda} R,\end{aligned} \tag{64}$$

where the terms in the last line describe the interaction of the Abelian $U(1)_Y$ tensor fields $B_\mu, B_{\mu\lambda}, \ldots$ with the right-handed sector of new leptons.

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References

[1] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).
[2] G. Savvidy, Phys. Lett. B **625**, 341 (2005), http://www.inp.demokritos.gr/~savvidy/Savvidis_2005.ppt.
[3] G. Savvidy, Generalization of Yang-Mills theory: Non-Abelian tensor gauge fields and higher-spin extension of standard model, arXiv:hep-th/0505033.

[4] G. K. Savvidy, Int. J. Mod. Phys. A **19**, 3171–3194 (2004).
[5] G. K. Savvidy, Phys. Lett. B **552**, 72 (2003).
[6] G. Savvidy, Phys. Lett. B **615**, 285 (2005).
[7] M. Fierz, Helv. Phys. Acta. **12**, 3 (1939).
M. Fierz and W. Pauli, Proc. Roy. Soc. A **173**, 211 (1939).
[8] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[9] H. Yukawa, Phys. Rev. **77**, 219 (1950).
M. Fierz, Phys. Rev. **78**, 184 (1950).
[10] J. Schwinger, Particles, Sourses, and Fields (Addison-Wesley, Reading, MA, 1970).
[11] S. Weinberg, Phys. Rev. **133**, B1318 (1964).
[12] S. J. Chang, Phys. Rev. **161**, 1308 (1967).
[13] L. P. S. Singh and C. R. Hagen, Phys. Rev. D **9**, 898 (1974).
[14] L. P. S. Singh and C. R. Hagen, Phys. Rev. D **9**, 910 (1974).
[15] C. Fronsdal, Phys. Rev. D **18**, 3624 (1978).
[16] J. Fang and C. Fronsdal, Phys. Rev. D **18**, 3630 (1978).
[17] J. Fang and C. Fronsdal, J. Math. Phys. **20**, 2264 (1979).
[18] F. A. Berends, G. J. H Burgers, and H. Van, Dam, Nucl. Phys. B **260**, 295 (1985).
[19] A. K. Bengtsson, I. Bengtsson, and L. Brink, Nucl. Phys. B **227**, 31 (1983).
[20] A. K. Bengtsson, I. Bengtsson, and L. Brink, Nucl. Phys. B **227**, 41 (1983).
[21] G. K. Savvidy, Symplectic and Large-N Gauge Theories, in: Proceedings of Vacuum Structure in Intense Fields, Cargese, 1990, v. 225 (Plenum Press, New York, 1991), p. 415; Renormalization of Infinite Dimensional Gauge Symmetries, Minnesota University and Yerevan Physics Institute Report TPI-MINN-90-2-T (1990), 18 pp.

