Hadron Structure
and
QCD

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Asymptotic Freedom of non-Abelian Tensor Gauge Fields.

Proton Structure and Tensor Gluons.

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1. Phys. Lett. B732 (2014) 150

2. ArXiv: 1310.0856

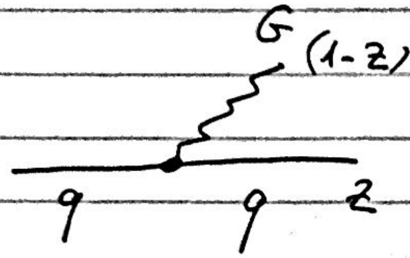
3. ArXiv: 1406.5334

1. Splitting Functions.
2. Generalization of DGLAP Equation
3. Momentum Sum Rules, Regularization
4. Callan-Simanzik beta function
5. Unification of Coupling Constants.
6. Conclusion.

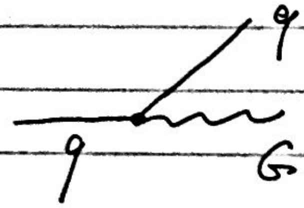
DGLAP Equation. Splitting Functions.

$$\begin{cases} \Delta q = P_{qq} \otimes q + P_{qG} \otimes G \\ \Delta G = P_{Gq} \otimes q + P_{GG} \otimes G \end{cases}$$

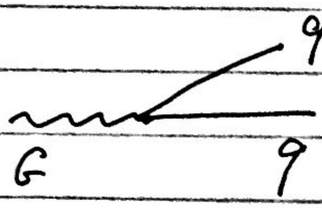
$$P_{qq} = C_2(R) \frac{1+z^2}{1-z}$$



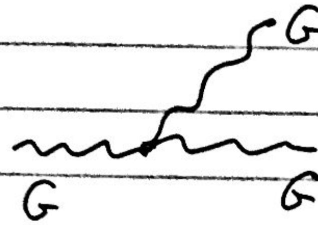
$$P_{Gq} = C_2(R) \frac{1+(1-z)^2}{z}$$



$$P_{qG} = T(R)\left[z^2 + (1-z)^2\right]$$

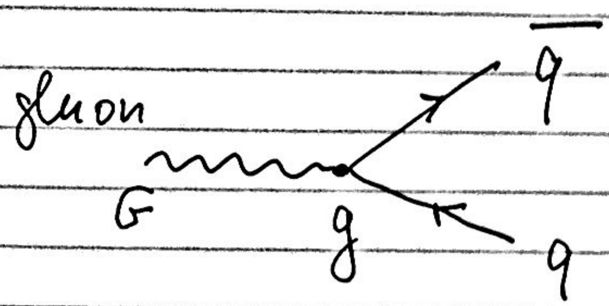


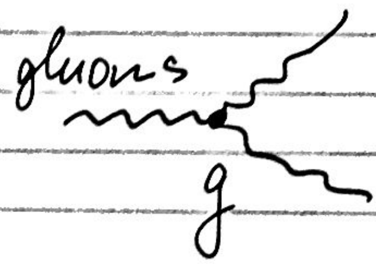
$$P_{GG} = C_2(G)\left[\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)}\right]$$



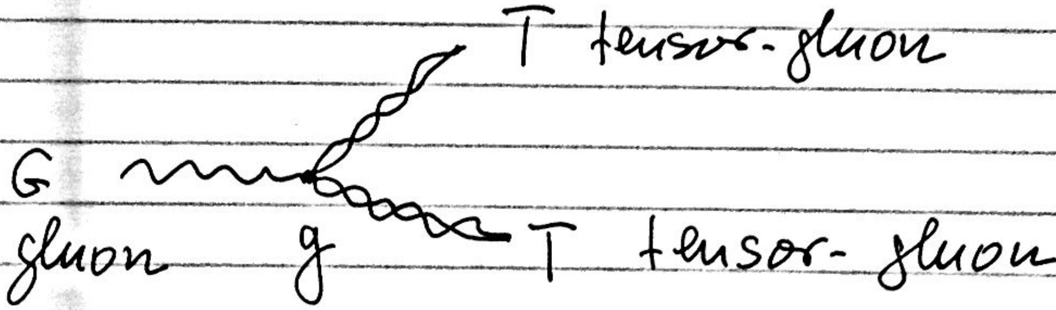
Non-Abelian Tensor

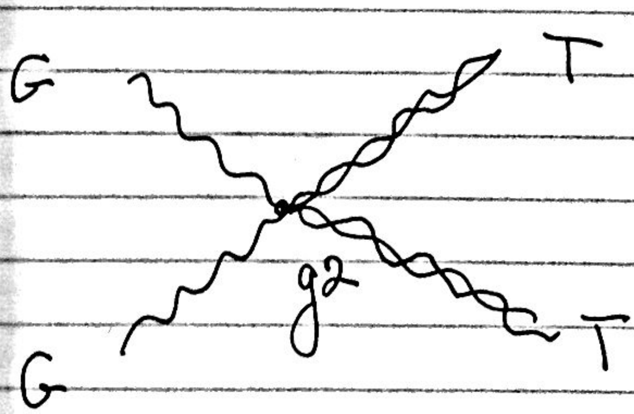
Gauge Bosons of QCD



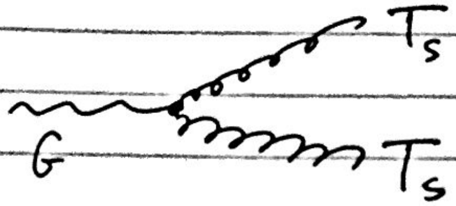


New interactions



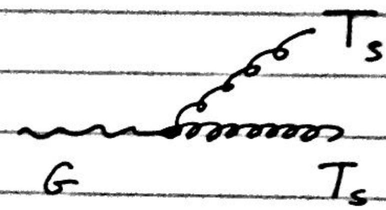


New Interaction Vertex

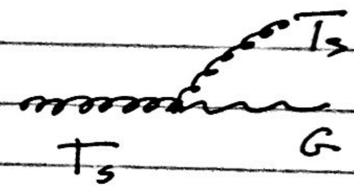


$$q(x,t)\,,\quad G(x,t)\,,\quad T_s(x,t)$$

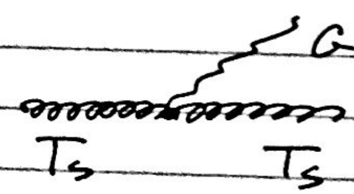
$$P_{TG} = C_2(G)\left[\frac{z^{2s+1}}{(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z^{2s-1}}\right]$$



$$P_{GT} = C_2(G)\left[\frac{1}{z(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z}\right]$$



$$P_{TT} = C_2(G)\left[\frac{1}{(1-z)z^{2s-1}} + \frac{z^{2s+1}}{1-z}\right]$$



$$G(x,t) + \sum_s T_s(x,t)$$

2. Extension of DGLAP Equation

$$\begin{cases} \Delta q = P_{qq} \otimes q + P_{qG} \otimes G \\ \Delta G = P_{Gq} \otimes q + P_{GG} \otimes G + P_{GT} \otimes T \\ \Delta T = P_{Tq} \otimes q + P_{TG} \otimes G + P_{TT} \otimes T \end{cases}$$

Momentum sum rules

$$\begin{cases} \int_0^1 dz\, z \left[P_{qq} + P_{Gq} \right] = 0 \\ \int_0^1 dz\, z \left[2n_f P_{qG} + P_{GG} + \sum_s P_{T_s G} \right] = 0 \\ \int_0^1 dz\, z \left[P_{GT_r} + \sum_s P_{T_s T_r} \right] = 0 \end{cases}$$

Callan-Simanzic beta function

$$\int_0^1 dz\, z \left[2n_f P_{qG}(z) + P_{GG}(z) + \sum_s P_{T_sG}(z) + b_1 \delta(z-1) \right] =$$

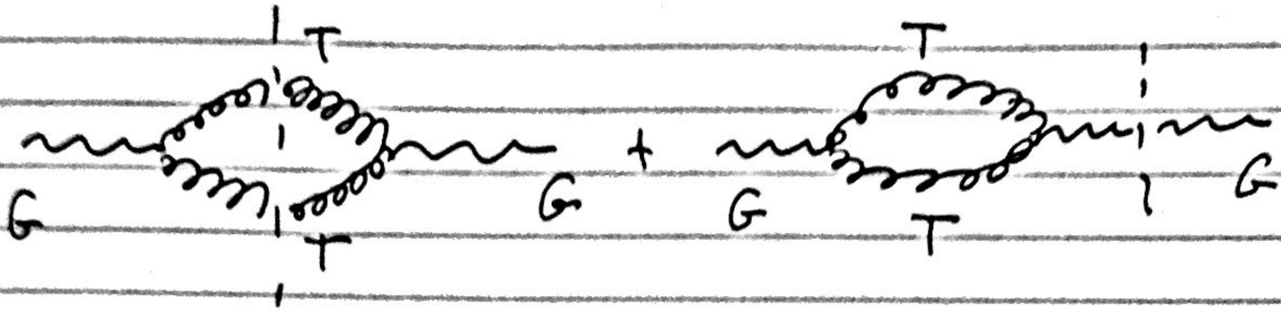
$$= \int_0^1 dz\, z \Bigg[2n_f T(R) \left(z^2 + (1-z)^2 \right) + C_2(G) \left(\frac{1}{z(1-z)} + \frac{z^4}{z(1-z)} + \frac{(1-z)^4}{z(1-z)} \right)$$

$$+ C_2(G) \sum_s \left(\frac{z^{2s+1}}{(1-z)^{2s-1}} + \frac{(1-z)^{2s+1}}{z^{2s-1}} \right) \Bigg] + b_1 = 0$$

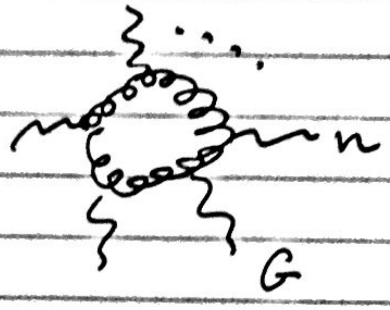
$$b_1 = -\frac{2n_f}{3} T(R) + \frac{11}{6} C_2(G) +$$

$$+ \sum_{s=2}^{\infty} \frac{12s^2 - 1}{6} C_2(G) \;;$$

$$\alpha_s(t) = \frac{\alpha_s}{1 + b_1 \alpha_s \ln Q^2/Q_0^2}$$



Momentum sum rules implicitly comprises unitarity.

$$\mathcal{L}^{(1)} = \sum_n$$ 

$$K_0^2 = (2n+1+2s)\, gH + K_\parallel^2$$

$$\mathcal{L}^{(1)} = \frac{H^2}{2} + \frac{(gH)^2}{4\pi}\, b_1 \left[\ln\frac{gH}{\mu^2} - \frac{1}{2}\right]$$

$$b_1 = -\frac{2C_2(G)}{\pi}\,\zeta\left(-1, \frac{2s+1}{2}\right) = \frac{12s^2-1}{12\pi}\, C_2(G)$$

$$\zeta(p,q) = \frac{1}{\Gamma(p)}\int_0^\infty dt\, t^{-1+p}\, \frac{e^{-qt}}{1-e^{-t}}$$

$$\zeta(-1,q) = -\frac{1}{2}\left(q^2 - q + \frac{1}{6}\right).$$

Conformal Invariance

$$b_1 = \frac{1}{2\pi}\sum_s \frac{12s^2-1}{6} C_2(G)$$

summation is over all spins of free spectrum:

$$\begin{array}{l}
\pm 1 \\
\pm 2,\ 0 \\
\pm 3,\ \pm 1,\ \pm 1 \\
\pm 4,\ \pm 2\ \pm 2\ \ 0 \\
\pm 5,\ \pm 3\ \pm 3\ \ \pm 1\ \pm 1 \\
\pm 6,\ \pm 4\ \pm 4\ \ \pm 2\ \pm 2\ 0 \\
\cdots
\end{array}$$

$$b_1 = C_2(G)\left[\sum_{s=1}^{\infty} \frac{12s^2-1}{12\pi} + \sum_{s=0}^{\infty} \frac{12s^2-1}{12\pi} + \cdots\right] =$$

$$= C_2(G)\left[\frac{1}{\pi}\zeta(-2) - \frac{1}{12\pi}\zeta(0) - \frac{1}{12\pi} + \cdots\right] =$$

$$= C_2(G)\left[\frac{1}{24\pi} - \frac{1}{12\pi} + \frac{1}{24\pi} + \cdots\right] = 0$$

Unification of Coupling Constants.

$$\left\{ \frac{1}{\alpha_i(M)} = \frac{1}{\alpha_i(\mu)} + 2b_i \ln\frac{M}{\mu} \qquad i=1,2,3 \qquad (A) \right.$$

$SU(3)_c \times SU(2)_L \times U(1)_{el}$ Contribution of $s=2$

$$2b = \frac{58\,C_2(G) - 4n_f\,T(R)}{6\pi}$$

$$2b_3 = \frac{54}{2\pi}\ ; \quad 2b_2 = \frac{1}{2\pi}\frac{104}{3}\ ; \quad 2b_1 = -\frac{4}{2\pi}$$

Solution of system (A)

$$\ln\frac{M}{\mu} = \frac{\pi}{58}\left(\frac{1}{\alpha_{el}(\mu)} - \frac{8}{3}\frac{1}{\alpha_s(\mu)}\right)$$

$$\alpha_{el}(M_Z) = \frac{1}{128} \qquad \alpha_s(M_Z) = \frac{1}{10} \ ; \quad \mu = M_Z$$

$$M \sim 40\,\text{TeV}$$