# The Nuclear Magnetic Octupole Moment

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# Abstract

The nuclear magnetic octupole moment is revisited as a potentially useful observable for nuclear structure studies. The magnetic octupole moment, *Ω*, is examined in terms of the nuclear collective model including weak and strong coupling. Single-particle formulation is additionally considered in the overall comparison of theoretical predictions with available experimental data. The symmetry of the nuclear force in mirror nuclei is also examined in terms of the magnetic octupole moment isoscalar and isovector terms.

# Motivation

A systematic study of octupole moments will:

- Help place constraints on the poorly known isoscalar part of nuclear spin-spin forces [1].
- Reduce the uncertainty of nuclear Schiff moments [2].

The experimental data that were collected from literature are about to be publicly available soon in the nuclear moments database, NUMOR [3,4]. This assemble, to our knowledge, is the most complete collection of *Ω* measurements.

# Mathematical Definition

The nuclear magnetic octupole moment $\Omega$ is defined as $\Omega = -M3$, where $M3$ is the expectation value of the corresponding operator [5]:

$$M_k^{\mu} = \mu_N \int \Psi^* (\nabla\, r^k C_{\mu}^{(k)}(\theta, \varphi) \cdot \left( g_l \frac{2}{k+1} \boldsymbol{L} + g_s \boldsymbol{S} \right) \Psi \, dv \qquad (1)$$

where the integral is over the nuclear volume $v$ and

$$C_{\mu}^{(k)}(\theta, \varphi) = \left( \frac{4\pi}{2k+1} \right)^{1/2} Y_{k\mu}(\theta, \varphi) \qquad (2)$$

is the multipole tensor operator of order $k$ with parity $(-1)^k$. This quantity vanishes for even $k$ and $k > 2I$. So, it becomes clear that $\Omega \neq 0$ for values $I > 3/2$ [6].

# Calculation of $\Omega$

The main problem with the calculation of the $\Omega$ is to find the appropriate wavefunctions to describe the nucleus. Consequently, $\Omega$ can be deduced using various models, such as:

- Single-particle model
- Collective model

# Single-particle model

In the extreme single-particle model $\Omega$ is expressed as [5]:

$$\Omega = +\mu_N \frac{3}{2}\frac{(2I-1)}{(2I+4)(2I+2)}\langle r^2\rangle \times \begin{cases} (I+2)[(I-{}^{3}/_{2})g_l + g_s] & I = l + {}^{1}/_{2} \\ (I-1)[(I+{}^{5}/_{2})g_l - g_s] & I = l - {}^{1}/_{2} \end{cases} \qquad (3)$$

It should not be trusted for more than a rough estimation of the value of $\Omega$, as the correct values depend strongly on nuclear many-body effects, more specifically on core polarization mediated by the nucleon spin-spin interaction [1,7].

# Strong coupling case

The form of the operator that acts on the particles is given by Eq. 1. The action of this operator should be summed over all valence nucleons.

As for the operator that acts on the core of the nucleus, considering quadrupole deformation ($\lambda = 2$) and using the strong coupling wave functions described in Ref. [8] we obtain the following result for the octupole magnetic moment:

$$\Omega = P_3\left(\Omega_{sp} + \Omega_0\right) \qquad (4)$$

where $P_3$ is given by:

$$P_3 = \frac{2I(2I-1)(2I-2)}{(2I+2)(2I+3)(2I+4)} \qquad (5)$$

# Strong coupling case

and $\Omega_0$ is given by:

$$\Omega_0 = -\frac{30}{49}\sqrt{\frac{2}{3}}g_R R_0^2 \qquad (6)$$

Eq.4 is correct for $I > {}^3/_2$. When $I = {}^3/_2$ the octupole moment is expressed as:

$$\Omega \simeq \frac{19}{35}\Omega_{sp} + \frac{1}{35}\Omega_0 \qquad (7)$$

# Weak coupling case

For the weak coupling case we can use perturbation theory considering the interaction of the angular momentum of the surface with the angular momentum of the particles as a small perturbation.

$$\Omega = \Omega_p + \Omega_c \tag{8}$$

Making some assumptions we get the simplified versions:

$$\Omega_c = -\frac{45}{224\pi} g_R R_0^2 \frac{k^2}{\hbar\omega C} \frac{(2I-1)(I-1)}{I(I+1)^2} \tag{9}$$

$$\Omega_p = \Omega_{sp}\left[1 + \frac{5}{32\pi}\frac{k^2}{\hbar\omega C}(\digamma_1 - \digamma_2)\right] \tag{10}$$

where

$$\digamma_1 = \frac{4I^2(I+1)^2 - 75I(I+1) + 234}{4I^2(I+1)^2} \quad \text{and} \quad \digamma_2 = \frac{(2I-1)(2I+3)}{4I(I+1)}$$

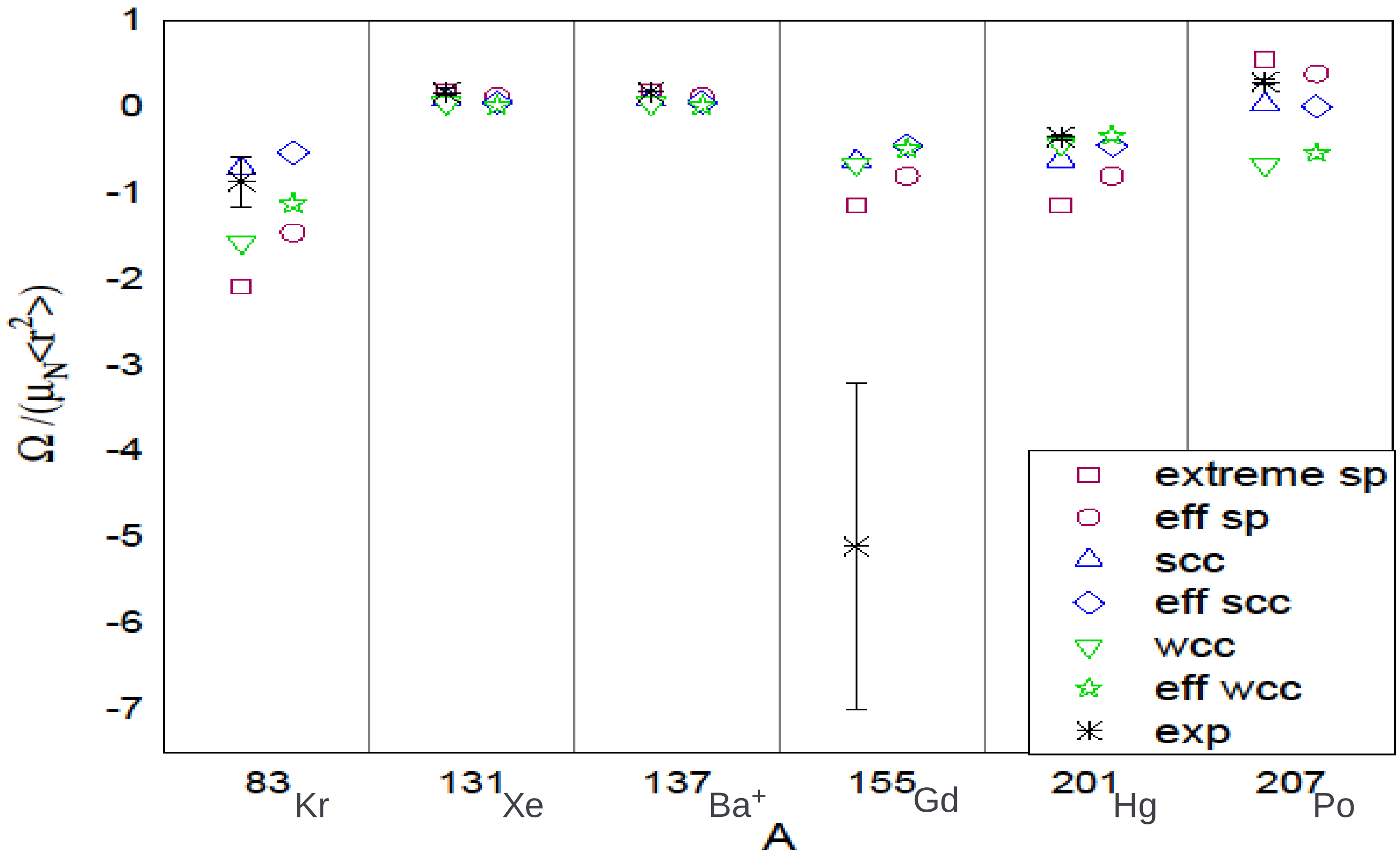


**Figure 1**: Visual comparison of experimental results *(exp)* for Ω and the predictions of extreme and effective *(eff)* single particle model *(sp, magenta)*, strong *(scc, blue)* and weak *(wcc, green)* coupling case for nuclei with an unpaired neutron, respectively. For better visualization, the points of *eff sp*, *eff scc* and *eff wcc* have been moved slightly to the right. The case of $^{173}$Yb is not presented in the diagram. For some isotopes the error bars are smaller than the size of the plotted points. Please note that the x-axis is not linear.

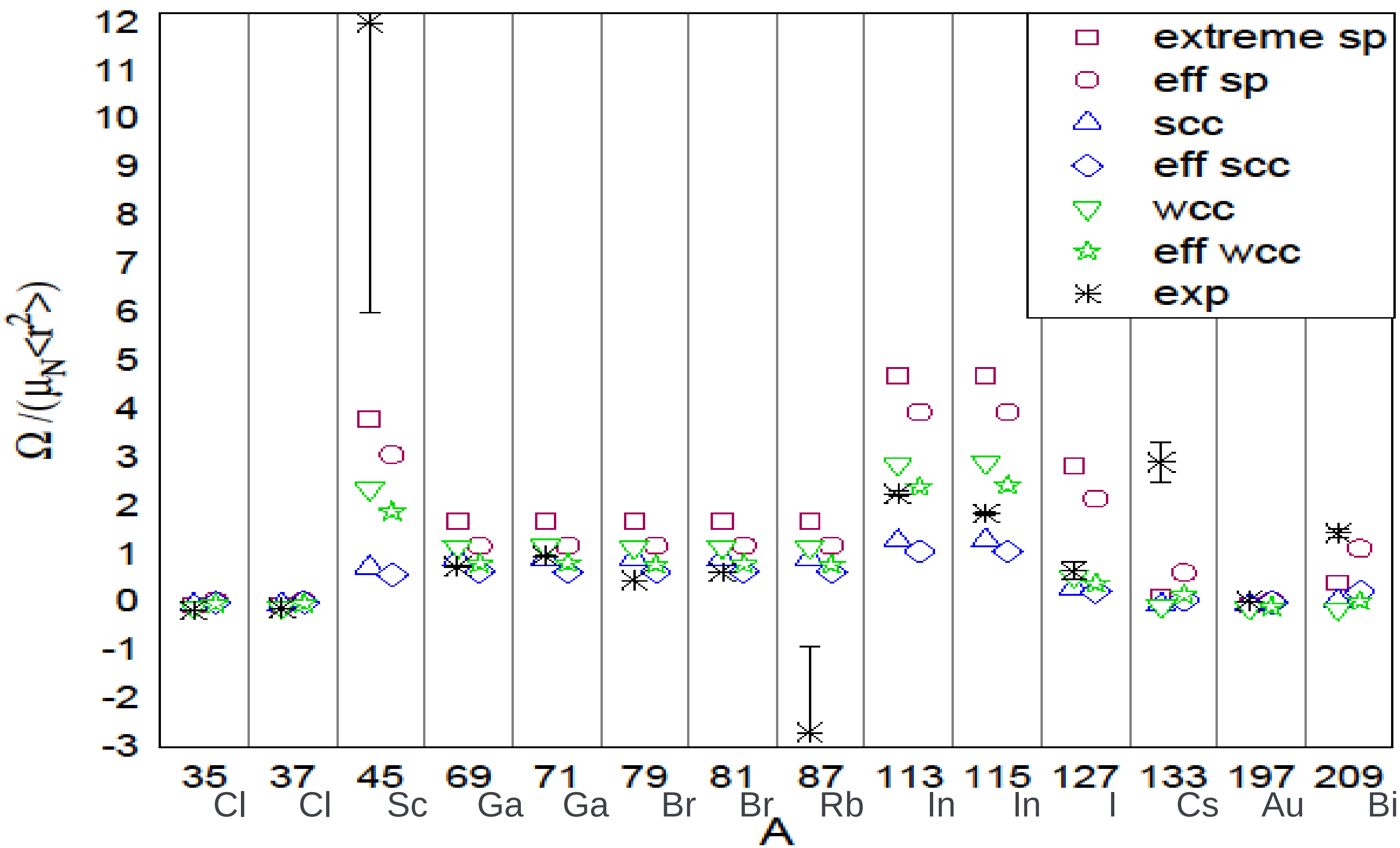


**Figure 2**: Visual comparison of experimental results *(exp)* for Ω and the predictions of extreme and effective *(eff)* single particle model *(sp, magenta)*, strong *(scc, blue)* and weak *(wcc, green)* coupling case for nuclei with an unpaired proton, respectively. For better visualization, the points of *eff sp*, *eff scc* and *eff wcc* have been moved slightly to the right. For some isotopes the error bars are smaller than the size of the plotted points. The values for $^{79}$Br and $^{81}$Br have no cited uncertainty in literature. Please note that the x-axis is not linear.

# Conclusions

- The collective model seems more favorable.
- Both strong and weak coupling case predict correctly the order of magnitude of $\Omega$ for the majority of nuclei with spin $I = l + 1/2$.
- General trend of the values of $\Omega$ to fall inside the Schmidt lines of extreme single-particle model.
- The effective weak coupling case gains ground in the theoretical description over the strong coupling case, challenging the conclusion drawn by Suekane and Yamaguchi.

The majority of the isotopes are in regions where their even-even cores are weakly collective.

For odd-neutron nuclei the expected model based on $R_{4/2}$ ratio has only occasionally predicted the correct order of magnitude for $\Omega$.

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