The Case for Nonlocal Modifications of Gravity

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2018年3月26日

Why change GR?
➔ It needs too much “dark” stuff

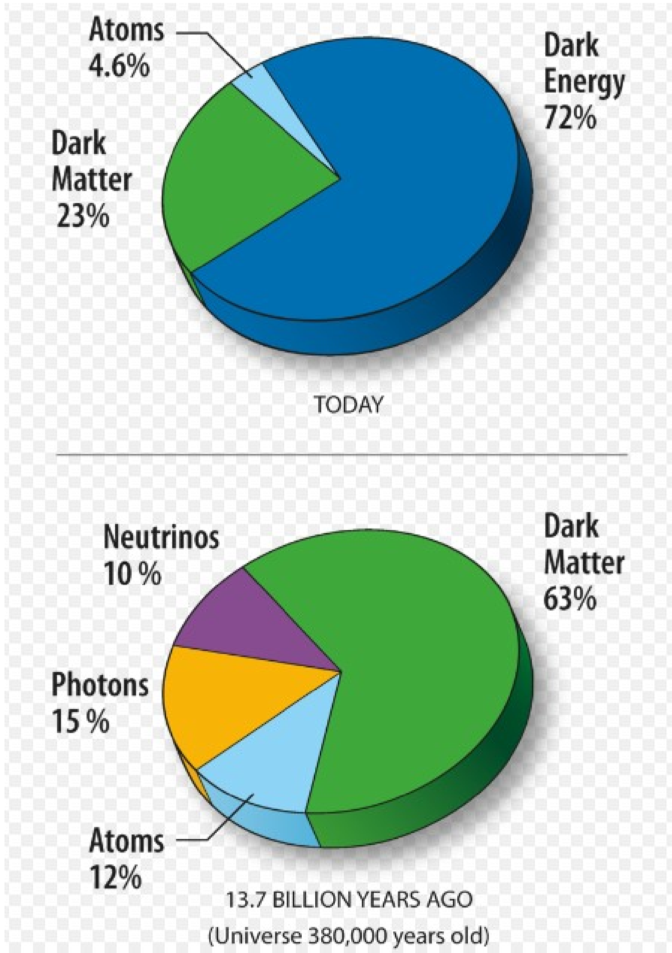
1. Dark Matter binds cosmic structures

   Galaxies & clusters

2. Dark Energy drives cosmic acceleration

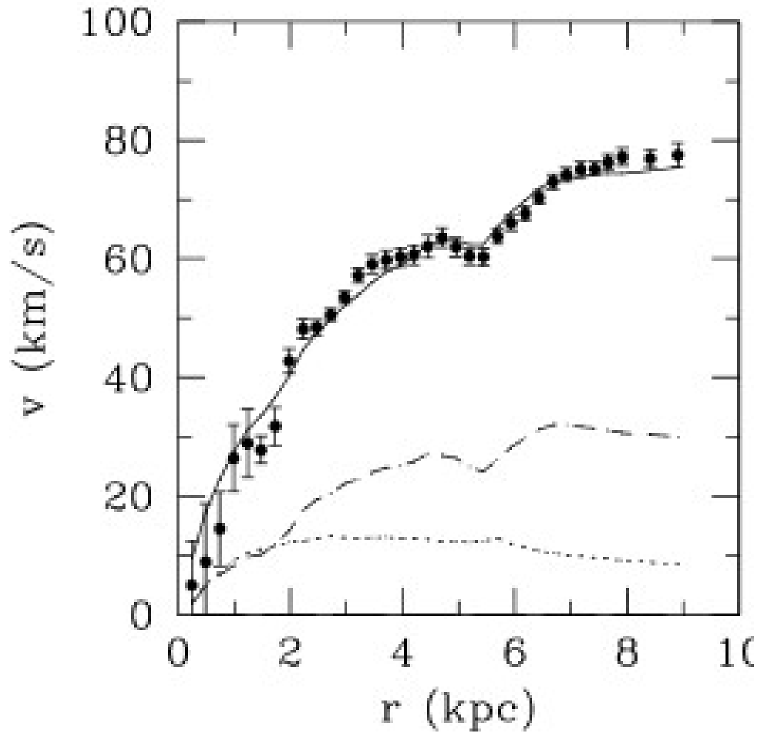
   a) Now

   b) Primordial inflation



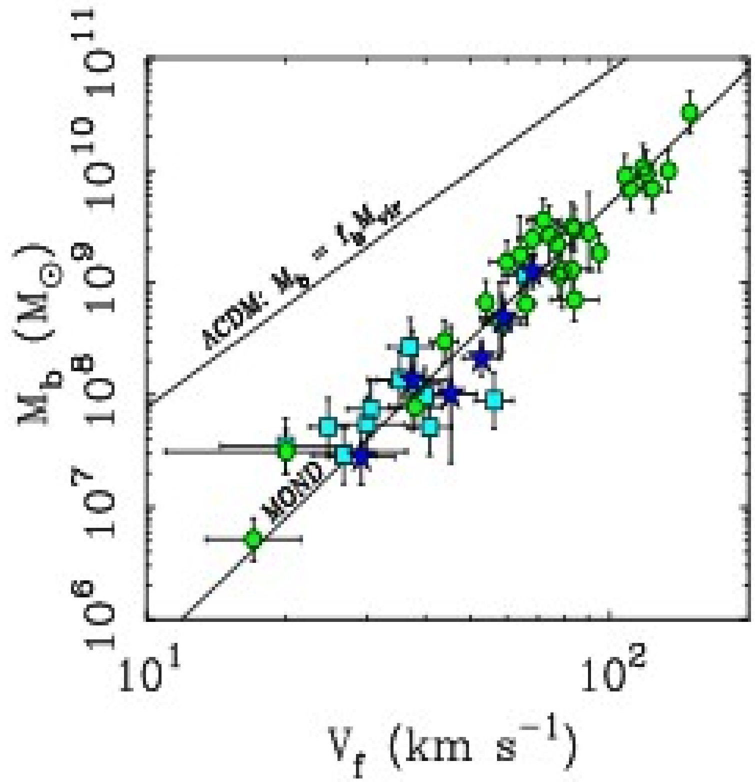
Dark Matter can be made to work, but

- Still no direct detection
  - XENON1T & PandaX-II null
  - Nothing at LHC either
- Unexplained regularities
  - Next slide
- Incorrect Distributions
  - Observation ➔ $\rho(r) \sim 1/r^2$
  - Simulations ➔ $\rho(r) \sim 1/r^3$
- Why is the baryonic ``tail'' wagging the CDM dog?
  - $M_c \sim 5.3 \times M_b$



Observed Regularities in rotationally supported systems

- Baryonic Tully-Fisher Relation:
  - Asymptotic $v^4 = a_0 GM$
  - $a_0 \sim 1.2 \times 10^{-10}\, {}^{m}/_{s^2}$
- Milgrom's Law:
  - Start needing DM for $g(r) < a_0$
- Freeman's Law:
  - Surface density $\Sigma < \frac{a_0}{G}$
- Sancisi's Law:
  - Bumps trace baryons
- Similar regularities for pressure-supported systems
  - All just accidents for DM!



Dark Energy can be made to work, but

- $\mathcal{L} = \frac{R\sqrt{-g}}{16\pi G} - \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$
  1) $+3H^2 = 8\pi G\left[\frac{1}{2}\dot{\varphi}_0^2 + V(\varphi_0)\right]$
  2) $-2\dot{H} - 3H^2 = 8\pi G\left[\frac{1}{2}\dot{\varphi}_0^2 - V(\varphi_0)\right]$
- Given $H(t)$ ➔ Reconstruct $V(\varphi)$ to support it
  - (1+2) ➔ $-2\dot{H} = 8\pi G\dot{\varphi}_0^2$ ➔ $\varphi_0(t) = \varphi_i \pm \int_{t_i}^{t} dt' \sqrt{\frac{-2\dot{H}(t')}{8\pi G}}$
  - Rotate the graph ➔ gives $t(\varphi_0)$
  - (1−2) ➔ $2\dot{H} + 6H^2 = 16\pi G\, V(\varphi_0)$ ➔ $V(\varphi) = \frac{\dot{H}(t(\varphi)) + 3H^2(t(\varphi))}{8\pi G}$
- But who ordered that?
  - Why is $\varphi(t,\vec{x}) \sim \varphi_0(t)$ so spatially homogeneous?
  - Why is $G^2 V(\varphi_0) \sim 10^{-122}$ so small?
  - Why is there no 5th force?

$f(R)$ can give (2b) but not (2a) or (1)

- Unique solution for ΛCDM is $f(R) = R - 2\Lambda$
  - Dunsby et al, arXiv:1005.2205
  - Other $f(R)$ models show deviations at 0^{th} order!
  - New scalar DoF requires screening mechanism
  - And why acceleration now?
- Cannot give Tully-Fisher: $v^4 = a_0 GM(r) = \left[\frac{1}{2}c^2 r b'\right]^2$
  - $ds^2 = -[1+b(r)](cdt)^2 + [1+a(r)]r^2 dr^2 + r^2 d\Omega^2$
  - $\frac{\delta S}{\delta b} = \frac{c^4}{16\pi G}\left[\frac{c^2}{2a_0}\,\partial_r(r^2 b'^2)\right] - \frac{1}{2}r^2\rho = 0$ has **3** $\boldsymbol{\partial_r}$'s
  - $R = -b'' + \frac{2(a'-b')}{r} + \frac{2a}{r^2}$ ➔ $f(R)$ could give 2, or 4 ∂_r's

Choices for modifying gravity

1. Employ additional fields
   - E.g., Bekenstein's TeVeS
   - Is this less epicyclic than DM & DE?
2. Employ only the metric
   a) Only local, invariant, stable option is $R \rightarrow f(R)$
   b) So either retain locality & abandon invariance
      - For example, Horava gravity & massive gravity
      - But GR170817 says graviton speed is nearly c
   c) Or else retain invariance & abandon locality

Isaac Newton’s Take on Nonlocality

“that one body may act upon another at a distance thro’ a Vacuum, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it.”

Was Newton too Harsh?

- I don’t think so (others here disagree)
  - Nonlocal theories have extra initial value data
    - Ostrogradskian instabilities
  - Contrary statements use Euclidean momentum space
    - We live in Minkowski signature
    - Instability precludes temporal Fourier transform
    - Euclidean k-space assumes the result & is not valid
- I believe fundamental theory is local
  - But quantum effective field equations are nonlocal
  - And loops of massless quanta can give big IR effects
    - Gravitons are massless!

Flat EM with vacuum polarization

Macroscopic nonlocality can happen!

- Charge runs in the UV, not IR, because $m_e \neq 0$
  - $-\nabla^2 \left[\Phi(r) + \int_0^\infty dk \frac{k \sin(kr)}{2\pi^2 r} \chi_e(k) \widetilde{\Phi}(k)\right] = Q\delta^3(\vec{x})$
  - $\chi_e(k) = \delta\chi_e + \frac{4\alpha}{\pi}\int_0^1 dx\, x(1-x) \left\{\ln\left(\frac{2\Lambda}{m_e}\right) - 1 - \frac{1}{2}\ln\left[1 + x(1-x)\frac{k^2}{m_e^2}\right]\right\}$
  - $\chi_e(k) = -\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x) \ln\left[1 + x(1-x)\frac{k^2}{m_e^2}\right]$ vanishes at $k = 0$
  - Small r ➔ $\Phi(r) = \frac{Q}{4\pi r}\left[1 + \frac{2\alpha}{3\pi}\ln\left(\frac{1}{m_e r}\right) + \cdots\right]$
- But $m_e = 0$ ➔ running in both UV & IR
  - $\chi_e(k) = -\frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x) \ln[x(1-x)k^2 R^2]$ vanishes for $k \sim \frac{1}{R}$
  - All r ➔ $\Phi(r) = \frac{Q}{4\pi r}\left[1 + \frac{2\alpha}{3\pi}\ln\left(\frac{R}{r}\right) + \cdots\right]$
- Perturbation theory breaks down at large r!
  - RG valid for IR ➔ $\Phi(r) = \frac{Q}{4\pi} \times \left[1 - \frac{4\alpha}{3\pi}\ln\left(\frac{R}{r}\right)\right]^{-\frac{1}{2}}$

Inflation produces scalars & gravitons

- Recall the primordial power spectra
  - $\Delta^2_{\mathcal{R}}(k)$ (resolved to 3 digits!) $\Delta^2_h(k)$ (hope soon)
- Each wave vector $\vec{k}$ has growing occupation
  - Inflatons ➔ $N_s(t,k) = \frac{\pi \Delta^2_{\mathcal{R}}(k)}{4Gk^2} \times \varepsilon(t) a^2(t)$
  - Gravitons ➔ $N_h(t,k) = \frac{\pi \Delta^2_h(k)}{64Gk^2} \times a^2(t)$
- These particles interact!
  - With themselves & with other quanta

Inflationary gravitons alter EM & GR

- EM on de Sitter background (co-moving, $a(t) = \exp[Ht]$)
  - Photons get secular enhancement (arXiv:1408.1448)
    - $F_{0i}^{1\,loop}(t,k) \to \frac{1}{\pi} GH^2 \ln(a) \times F_{0i}^{tree}(t,k)$
  - Coulomb potential runs in the IR (arXiv:1308.3453)
    - $\Phi(r) = \frac{Q}{4\pi a r}\left[1 + \frac{G}{3\pi a^2 r^2} + \frac{1}{\pi} GH^2 \ln(aHr) + O(G^2)\right]$
- GR on de Sitter background (co-moving, $a(t) = \exp[Ht]$)
  - Gravitons get secular de-enhancement (arXiv:1307.1422)
    - $C_{0i0j}^{1\,loop}(t,k) \to -\frac{8}{\pi} GH^2 \ln(a) \times C_{0i0j}^{tree}(t,k)$
  - Potential screened for large r & t (arXiv:1510.03352 from φ's)
    - $\Phi(r) = -\frac{GM}{ar}\left[1 + \frac{G}{20\pi a^2 r^2} - \frac{GH^2}{10}\left(\frac{1}{3}\ln(a) + 3\ln(aHr)\right) + O(G^2)\right]$
- Perturbation theory breaks down for large r & t

Tedious criticisms from the "head-in-the-sand" faction

1) "Effects are gauge dependent"
   - Not true for arXiv:1510.03352 from scalars
   - Clear flat space antecedents
     - Pulsar timing & G/r^2 corrections
2) "IR gravitons have small curvature"
   - Not initially ➔ "Memory Effect" in flat space
   - Predictions falsified (arXiv:1606.02417)
     - Confusing IR divergences with IR effects
3) "Effects are not observable"
   - Measure nearby $q_1 = 0$ & $q_2 \neq 0$
4) "The calculations are difficult"
   - Grow up!

i	1	a	$\frac{1}{b-2}$	$\frac{(a-3)}{(b-2)^2}$
0	$+\frac{3}{4}$	$-\frac{3}{4}$	$-\frac{3}{2}$	$+\frac{3}{4}$
1	0	0	0	$+1$
2	0	0	0	0
3	0	0	$+3$	-2
4	$+\frac{17}{4}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$
5	-2	$+\frac{3}{2}$	$-\frac{3}{2}$	$+\frac{1}{2}$
Total	$+3$	0	0	0

More on the gauge issue

- Why the EFE's are gauge dependent
  - Effective field is disturbed by some *source*
  - And measured by some *observer*
  - Source & observer interact with QG!
- ArXiv:1708.06239 ➔ graviton correction to scalar force
  - $\mathcal{L}_{GF} = -\frac{1}{2a} F_\mu F^\mu$ $\qquad F_\mu = \partial^\nu h_{\mu\nu} - \frac{b}{2} \partial_\mu h^\nu{}_\nu$
  - $-iM_0^2(x;x') = C_0(a,b) \times \frac{G\partial^6}{4\pi^3} \left[\frac{\ln(\mu^2 \Delta x^2)}{\Delta x^2}\right]$
  - $C_0(a,b) = \frac{3}{4} \frac{(b-1)}{(b-2)^2} [(b-5) - (b-3)a]$ from $-\infty$ to $+\infty$
- QG correlations from source & observer cancel gauge dependence
  - Table on previous slide
- "Just because something is gauge dependent does not mean it is zero!"

Still no derivation . . .

- So models are purely phenomenological
- But inflationary origin explains two things
  1) There is an initial time, corresponding to when the QFT state was released; and
  2) We should expect modifications on large scales, not small scales, because inflation produces IR gravitons.

Guidance from Perturbation Theory

- What are the de Sitter factors of $\ln(a)$ generally?
  - $\frac{1}{\square} R \rightarrow -4 \ln\left(\frac{a(t)}{a_i}\right)$
- Advantages
  - Simplest nonlocal scalar (cf. effective field theory)
  - Dimensionless ➔ no mass parameters
  - Grows during inflation
  - Quiescent during radiation domination $(R = 0)$
  - Using R (or $R_{\mu\nu}$) ➔ GR radiation unchanged

Nonlocal Cosmology
(Late Time Acceleration)

- $\Delta\mathcal{L} = \frac{1}{16\pi G} R f\left(\frac{1}{\Box} R\right)\sqrt{-g}$
  - arXiv:0706.2151 with Deser
  - No dimensionful parameter needed
- Can choose $f(X)$ to reproduce ΛCDM
  - arXiv:0904.0961 with Deffayet
- Onset naturally delayed to very late times
  - $R = 0$ during Radiation Domination
  - $\frac{1}{\Box} R$ grows only logarithmically thereafter
- $\frac{1}{\Box} R$ changes sign from cosmology to bound systems
  - No compensation mechanism needed to preserve solar system
- Agrees with structure formation data better than GR
  - arXiv:1701.00434 and arXiv:1711.08759

Nonlocal MOND

arXiv:1106.4984 & 1405.0393 with Deffayet & Esposito-Farese

- $\Delta\mathcal{L} = \frac{1}{16\pi G} a_0^2 f_y(\text{Ƶ})\sqrt{-g}$
  - $\text{Ƶ}[g] = \frac{4}{a_0^2} g^{\mu\nu}\partial_\mu \left(\frac{1}{\Box} R_{\rho\sigma}u^\rho u^\sigma\right)\partial_\nu \left(\frac{1}{\Box} R_{\alpha\beta}u^\alpha u^\beta\right)$
- $ds^2 = -[1 + b(r)]dt^2 + [1 + a(r)]dr^2 + r^2 d\Omega^2$
  - $\text{Ƶ} = {b'^2}/{a_0^2} + \cdots$
- $f_y(\text{Ƶ}) = \frac{1}{2}\text{Ƶ} - \frac{1}{6}\text{Ƶ}^{3/2} + o(\text{Ƶ}^2) \to \frac{1}{2}\text{Ƶ}\exp\left[-\frac{1}{3}\sqrt{\text{Ƶ}}\right]$ for $\text{Ƶ} > 0$
  - $\frac{1}{2}\text{Ƶ}$ term cancels weak field GR term
  - $-\frac{1}{6}\text{Ƶ}^{3/2}$ gives the weak field MOND equation
  - Large $\text{Ƶ} > 0$ suppressed ➔ no change to solar system
- Cosmology has $\text{Ƶ} < 0$
  - Choose $f_y(\text{Ƶ})$ to almost reproduce ΛCDM (arXiv:1608.07858)
  - Structure formation problematic (arXiv:1804.01669)

Primordial Inflation

- What are the de Sitter factors of $\ln(a)$ generally?
  - L loops ➔ $[GH^2 \ln(a)]^L$
  - Perhaps $GH^2 \ln(a) \rightarrow \frac{1}{\square}\left(\frac{1}{12} G R R_{\rho\sigma} u^\rho u^\sigma\right)$?
- Advantages
  - Grows during inflation
  - Quiescent during radiation domination $(R = 0)$
  - Matter domination reactivates (but R_{00} sign changes)
  - Using R (or $R_{\mu\nu}$) ➔ GR radiation unchanged
- Why not localize? ➔ Ghosts!
  - $f\left(\frac{1}{\square} Q\right)\sqrt{-g} \leftrightarrow f(\phi)\sqrt{-g} - \partial_\mu \phi \partial_\nu \psi g^{\mu\nu}\sqrt{-g} - Q\psi\sqrt{-g}$
  - $A_\pm \equiv \phi \pm \psi$ ➔ $\partial_\mu \phi \partial_\nu \psi g^{\mu\nu} = \frac{1}{4}(\partial_\mu A_+ \partial_\nu A_+ - \partial_\mu A_- \partial_\nu A_-) g^{\mu\nu}$

Conclusions

- GR needs ``dark'' stuff on cosmic scales
  - DM & DE both ``work'', but are epicyclic
- $R \rightarrow f(R)$ only invariant, local, stable $g_{\mu\nu}$ model
  - Cannot replace DM or late DE but can describe inflation
- Nonlocal gravity is a new frontier
  - Can replace DM, late DE and inflation
- Not fundamental, rather (I believe) an EFT
  - Vacuum polarization from inflationary gravitons
  - Hence there is an initial time & only modifications at large scales
- Challenges
  - Derive it from 1st principles
  - Avoid ghosts, avoid modifying gravitational radiation
  - Get structure formation right